# REPORT No. 906

# DETERMINATION OF STRESSES IN GAS-TURBINE DISKS SUBJECTED TO PLASTIC FLOW AND CREEP

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## SUMMARY

*A finite-difference method previously presented for computing elastic stresses in rotating disks is extended to include the computation of the disk stresses when plastic flow and creep are considered. A finite-difference method is employed to eliminate numerical integration and to permit nontechnical personnel to make the calculations with a minimum of engineering supervision. Illustrative examples are included to facilitate explanation of the procedure by carrying out the computations on a typical gas-turbine disk through a complete running cycle.*

*The results of the numerical examples presented indicate that plastic flow markedly alters the elastic-stress distribution.*


## INTRODUCTION

With the advent of jet propulsion as a motive force for aircraft, the gas turbine has become an important source of power. In most machinery, design stresses are limited by the yield strength or the creep strength of the material employed, together with a certain factor of safety, and little or no analytical consideration is given to the occurrence of plastic flow under operating conditions. Gas-turbine disks, however, are required to operate under thermal gradients and centrifugal forces producing stresses that, in materials currently available, frequently exceed the yield strength, resulting in plastic flow. The interaction of plastic flow and creep, together with the variation of thermal gradients through a series of cycles consisting in starting, running, and stopping, can produce stress distributions and even failures that might not be anticipated on a basis of elastic-stress analysis.

A rapid routine method of elastic-stress analysis of rotating disks is presented in reference 1, which gives accurate values of the true stresses in disks provided that the yield strength of the material is not exceeded. The finite-difference method of reference 1 has been extended at the NACA Cleveland laboratory to include consideration of plastic flow and creep, which thus allows calculation of the true stresses in a gas-turbine disk and gives the variation of stress distribution with time. The handling of plastic flow is somewhat less routine than the calculation of the elastic stresses in that a repetitive trial procedure is required. With practice, the correct value can be obtained on the fourth or fifth trial. The computation of the effect of creep, although in procedure the same as the computation of plastic flow, is a direct calculation requiring no trial-and-error procedures. Because the method eliminates numerical integration, nontechnical personnel can make the calculations with a minimum of engineering supervision.

## SYMBOLS

The following symbols are used:

| | |
|---|---|
| $c$ | creep rate under stress $\sigma_e$, (in./(in.)(hr)) |
| $E$ | elastic modulus of disk material, (lb/sq in.) |
| $h$ | axial thickness of disk, (in.) |
| $R$ | ratio $\left(\frac{3\epsilon_p}{2\sigma_e}\right)$ |
| $r$ | radial distance, (in.) |
| $T$ | temperature, (°F) |
| $u$ | radial displacement, (in.) |
| $\alpha$ | coefficient of thermal expansion between actual temperature and temperature at zero thermal stress, (in./(in.)(°F)) |
| $\Gamma$ | total creep under stress $\sigma_e$, (in./in.) |
| $\Delta$ | plastic increment of strain, (in./in.) |
| $\Delta_r$ | plastic increment of strain in radial direction, (in./in.) |
| $\Delta_t$ | plastic increment of strain in tangential direction, (in./in.) |
| $\Delta T$ | temperature increment above temperature of zero thermal stress, (°F) |
| $\delta_r$ | creep increment in radial direction, (in./in.) |
| $\delta_t$ | creep increment in tangential direction, (in./in.) |
| $\epsilon$ | strain, (in./in.) |
| $\epsilon_p$ | plastic strain corresponding to stress $\sigma_e$ in tensile specimen, (in./in.) |
| $\epsilon_r$ | radial strain, (in./in.) |
| $\epsilon_t$ | tangential strain, (in./in.) |
| $\mu$ | Poisson's ratio |
| $\rho$ | mass density of disk material, ((lb)(sec$^2$)/in.$^4$) |
| $\sigma$ | stress, (lb/sq in.) |
| $\sigma_e$ | equivalent tensile stress, (lb/sq in.) |
| $\sigma_r$ | radial stress, (lb/sq in.) |
| $\sigma_t$ | tangential stress, (lb/sq in.) |
| $\sigma_y$ | proportional elastic limit, (lb/sq in.) |
| $\tau$ | time during which creep occurs, (hr) |
| $\omega$ | angular velocity, (radians/sec) |

The following supplementary subscripts are used for denoting values of the preceding symbols in connection with the finite-difference solution:

$n$ $n$th point station

$n$-1 ($n$-1)st point station

$a$ station at smallest disk radius considered (For disk with a central hole, this station is taken at the radius of the central hole; for a solid disk, this station is taken at a radius approximately 5 percent of the rim radius.)

$b$ station at rim of disk or base of blades

The following supplementary symbols denote combinations of the foregoing symbols:

$\left.\begin{array}{l} A_{r,n} \\ A_{t,n} \\ B_{r,n} \\ B_{t,n} \end{array}\right\}$ stress coefficients defined by equations $\sigma_{r,n}=A_{r,n}\sigma_{t,a}+B_{r,n}$, $\sigma_{t,n}=A_{t,n}\sigma_{t,a}+B_{t,n}$

$C_n \;=r_n h_n$

$C'_n=\dfrac{\mu_n}{E_n}+\dfrac{(1+\mu_n)(r_n-r_{n-1})}{2E_n r_n}$

$D_n \;=\dfrac{1}{2}(r_n-r_{n-1})h_n$

$D'_n=\dfrac{1}{E_n}+\dfrac{(1+\mu_n)(r_n-r_{n-1})}{2E_n r_n}$

$F_n \;=r_{n-1}h_{n-1}$

$F'_n=\dfrac{\mu_{n-1}}{E_{n-1}}-\dfrac{(1+\mu_{n-1})(r_n-r_{n-1})}{2E_{n-1}r_{n-1}}$

$G_n \;=\dfrac{1}{2}(r_n-r_{n-1})h_{n-1}$

$G'_n=\dfrac{1}{E_{n-1}}-\dfrac{(1+\mu_{n-1})(r_n-r_{n-1})}{2E_{n-1}r_{n-1}}$

$H_n \;=\dfrac{1}{2}\omega^2(r_n-r_{n-1})(\rho_n h_n r_n^2+\rho_{n-1}h_{n-1}r_{n-1}^2)$

$H'_n=\alpha_n\Delta T_n-\alpha_{n-1}\Delta T_{n-1}$

$K_n \;=\dfrac{F'_nD_n-F_nD'_n}{C'_nD_n-C_nD'_n}$

$K'_n=\dfrac{C_nF'_n-C'_nF_n}{C'_nD_n-C_nD'_n}$

$L_n \;=-\dfrac{G'_nD_n+G_nD'_n}{C'_nD_n-C_nD'_n}$

$L'_n=-\dfrac{C'_nG_n+C_nG'_n}{C'_nD_n-C_nD'_n}$

$M_n=\dfrac{D'_nH_n+D_n(H'_n-P'_n-Q'_n)}{C'_nD_n-C_nD'_n}$

$M'_n=\dfrac{C'_nH_n+C_n(H'_n-P'_n-Q'_n)}{C'_nD_n-C_nD'_n}$

($M_n$ and $M'_n$ are defined in reference 1 for the special case $P'_n=Q'_n=0$)

$$P'_n=\Delta_{r,n}\left(\frac{r_n-r_{n-1}}{2r_n}\right)+\Delta_{r,n-1}\left(\frac{r_n-r_{n-1}}{2r_{n-1}}\right)-\Delta_{t,n}\left(1+\frac{r_n-r_{n-1}}{2r_n}\right)+\Delta_{t,n-1}\left(1-\frac{r_n-r_{n-1}}{2r_{n-1}}\right)$$

$$Q'_n=\delta_{r,n}\left(\frac{r_n-r_{n-1}}{2r_n}\right)+\delta_{r,n-1}\left(\frac{r_n-r_{n-1}}{2r_{n-1}}\right)-\delta_{t,n}\left(1+\frac{r_n-r_{n-1}}{2r_n}\right)+\delta_{t,n-1}\left(1-\frac{r_n-r_{n-1}}{2r_{n-1}}\right)$$

## ANALYSIS OF PLASTIC FLOW AND CREEP

**Assumptions.**—Four assumptions are made in the subsequent analysis:

1. The disk material is linearly elastic up to a limiting stress value, called the proportional elastic limit, and above this limit plastic flow occurs.
2. All variables of material properties and operating conditions are symmetrical about the axis of rotation.
3. Axial stresses may be neglected and the radial and tangential stresses are uniform across the thickness of the disk.
4. Temperatures are uniform across the thickness of the disk.

**Outline of method.**—In any thin rotating disk, the complete stress state is defined when the two principal stresses, radial $\sigma_r$ and tangential $\sigma_t$, are known at every radius. Two equations relating these stresses to the radius are required to specify the stress distribution. The first of these equations can be determined from the conditions of equilibrium of an element of the disk and involves no elastic properties of the material. The second is derived from the compatibility conditions, which state the interrelation of radial and tangential strains. The compatibility conditions are dependent upon stress-strain phenomena and must therefore include any departure from linear elasticity. When modification to allow for any possible departure from Hooke's law is made, the compatibility conditions become true for any value of stress. The equation derived from the compatibility conditions thus modified, together with the equilibrium equation, is treated by the finite-difference method of reference 1, and similar equations are obtained. These equations result in additional terms in the final equations, which are used to modify the result of the elastic calculation.

Whenever stresses under discussion have been calculated by the method of reference 1 only, they will be referred to as "elastic stresses"; where plastic flow and creep have been taken into account, the stresses will be referred to as "plastic stresses."

**Derivation of method.**—The equilibrium equation, which applies to both the elastic and plastic conditions, is

$$\frac{d}{dr}(rh\sigma_r)-h\sigma_t+\rho\omega^2r^2h=0 \tag{1}$$

The elastic compatibility relations given in terms of the radial displacement are

$$\epsilon_r=\frac{du}{dr}=\frac{\sigma_r-\mu\sigma_t}{E}+\alpha\Delta T \tag{2}$$

and

$$\epsilon_t=\frac{u}{r}=\frac{\sigma_t-\mu\sigma_r}{E}+\alpha\Delta T \tag{3}$$

Equations (2) and (3) must be modified to include consideration of plastic flow. When a material is stressed beyond the proportional elastic limit, the strain in the material is different from that indicated by Hooke's law. The strain under such a load may be considered as being made up of two components, one elastic as predicted by the laws of elasticity and one an increment of strain due to the flow that occurs. Rewriting equations (2) and (3) on this basis gives

$$\epsilon_r=\frac{du}{dr}=\frac{\sigma_r-\mu\sigma_t}{E}+\alpha\Delta T+\Delta_r \tag{4}$$

$$\epsilon_t=\frac{u}{r}=\frac{\sigma_t-\mu\sigma_r}{E}+\alpha\Delta T+\Delta_t \tag{5}$$

Similarly, any creep that occurs represents an additional departure from elastic behavior, which further modifies equations (2) and (3) to

$$\epsilon_r=\frac{du}{dr}=\frac{\sigma_r-\mu\sigma_t}{E}+\alpha\Delta T+\Delta_r+\delta_r \tag{6}$$

$$\epsilon_t=\frac{u}{r}=\frac{\sigma_t-\mu\sigma_r}{E}+\alpha\Delta T+\Delta_t+\delta_t \tag{7}$$

When the parameter $u$ is eliminated as in reference 1,

$$\frac{d}{dr}\left(\frac{1}{E}\sigma_t-\frac{\mu}{E}\sigma_r+\alpha\Delta T+\Delta_t+\delta_t\right)=\frac{1+\mu}{Er}(\sigma_r-\sigma_t)+\frac{\Delta_r-\Delta_t}{r}+\frac{\delta_r-\delta_t}{r} \tag{8}$$

Applying the finite-difference method to equations (1) and (8) and using the notation introduced in the section Symbols gives

$$C_n\sigma_{r,n}-D_n\sigma_{t,n}=F_n\sigma_{r,n-1}+G_n\sigma_{t,n-1}-H_n \tag{9}$$

and

$$C'_n\sigma_{r,n}-D'_n\sigma_{t,n}=F'_n\sigma_{r,n-1}-G'_n\sigma_{t,n-1}+H'_n-P'_n-Q'_n \tag{10}$$

The solution of the equations is facilitated by the substitution of the stress coefficients $A_{r,n}$, $A_{t,n}$, $B_{r,n}$, and $B_{t,n}$ into equations (9) and (10). Proceeding as in reference 1 results in the equations

$$\left.\begin{aligned}C_nA_{r,n}-D_nA_{t,n}-F_nA_{r,n-1}-G_nA_{t,n-1}=0\\ C'_nA_{r,n}-D'_nA_{t,n}-F'_nA_{r,n-1}+G'_nA_{t,n-1}=0\\ C_nB_{r,n}-D_nB_{t,n}-F_nB_{r,n-1}-G_nB_{t,n-1}+H_n=0\\ C'_nB_{r,n}-D'_nB_{t,n}-F'_nB_{r,n-1}+\\ G'_nB_{t,n-1}-H'_n+P'_n+Q'_n=0\end{aligned}\right\} \tag{11}$$

All but the last of equations (11) and equations (15) of reference 1 are identical. When equations (11) are solved for $A_{r,n}, A_{t,n}, B_{r,n}$, and $B_{t,n}$,

$$\left.\begin{aligned}A_{r,n}&=K_nA_{r,n-1}+L_nA_{t,n-1}\\ A_{t,n}&=K'_nA_{r,n-1}+L'_nA_{t,n-1}\\ B_{r,n}&=K_nB_{r,n-1}+L_nB_{t,n-1}+M_n\\ B_{t,n}&=K'_nB_{r,n-1}+L'_nB_{t,n-1}+M'_n\end{aligned}\right\} \tag{12}$$

The symbols $K_n$, $K'_n$, $L_n$, and $L'_n$ have the same meaning as in reference 1. The $M_n$ and $M'_n$ terms are now defined as

$$M_n=\frac{D'_nH_n+D_n(H'_n-P'_n-Q'_n)}{C'_nD_n-C_nD'_n} \tag{13}$$

$$M'_n=\frac{C'_nH_n+C_n(H'_n-P'_n-Q'_n)}{C'_nD_n-C_nD'_n} \tag{13a}$$

The elastic case of reference 1 thus becomes a special case of the more general problem in which $P'_n$ and $Q'_n$ are both zero.

**Evaluation of plastic terms.**—In order to apply the finite-difference method to problems involving plastic flow, a relation between stresses and strains in the plastic region must be established. In reference 2, a numerical-integration method for computing disk stresses is presented in which elongation is assumed to proceed at constant stress when the proportional limit is reached. References 3 and 4 present equations for the plastic relation of stress to strain based on the maximum distortion theory. Relations can be derived from the equations given in reference 4, which form a convenient means of finding the plastic increments corresponding to the stresses present in the disk. Rewriting these equations in the notation of this report and letting the subscripts 1, 2, and 3 denote the three principal directions in the most general case give

$$\left.\begin{aligned}\Delta_1&=\frac{R}{3}\left[(\sigma_1-\sigma_2)+(\sigma_1-\sigma_3)\right]\\ \Delta_2&=\frac{R}{3}\left[(\sigma_2-\sigma_1)+(\sigma_2-\sigma_3)\right]\\ \Delta_3&=\frac{R}{3}\left[(\sigma_3-\sigma_1)+(\sigma_3-\sigma_2)\right]\end{aligned}\right\} \tag{14}$$

$$\sigma_e=\frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} \tag{15}$$

where the ratio $R$ is defined in terms of the corresponding uniaxial stress $\sigma_e$ and plastic strain $\epsilon_p$ in a tensile specimen by the relation

$$R=\frac{3\epsilon_p}{2\sigma_e} \tag{16}$$

When equations (14), (15), and (16) are reduced to the biaxial condition, which is assumed to prevail in the disk (that is, $\sigma_3=0$), and the finite-difference notation is introduced

$$\left.\begin{aligned}\Delta_{r,n}&=\frac{R}{3}\left(\sigma_{r,n}-\sigma_{t,n}+\sigma_{r,n}\right)\\ \Delta_{t,n}&=\frac{R}{3}\left(\sigma_{t,n}-\sigma_{r,n}+\sigma_{t,n}\right)\end{aligned}\right\} \tag{17}$$

$$\sigma_{e,n}=\sqrt{\sigma_{r,n}^2-\sigma_{r,n}\sigma_{t,n}+\sigma_{t,n}^2} \tag{18}$$

and

$$R=\frac{3\epsilon_{p,n}}{2\sigma_{e,n}} \tag{19}$$

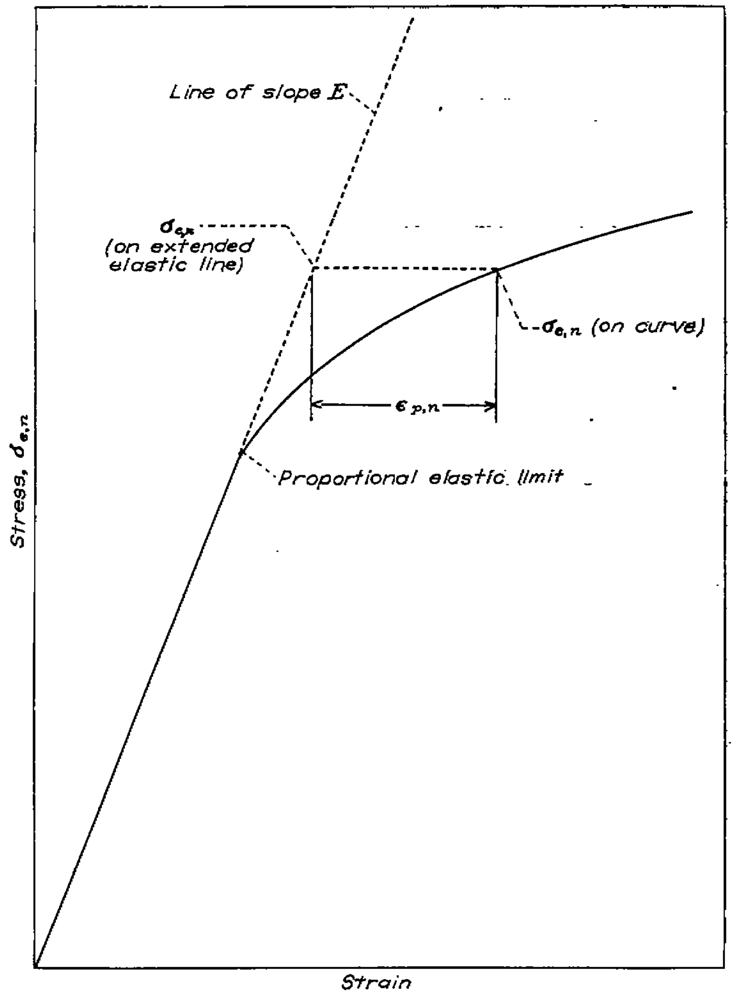



FIGURE 1.—Uniaxial stress-strain curve showing relation between stress $\sigma_{e,n}$ and plastic strain $\epsilon_{p,n}$.

Substituting equation (19) into equations (17) gives

$$\left.\begin{aligned}\Delta_{r,n}&=\frac{\epsilon_{p,n}}{2\sigma_{e,n}}\,(2\sigma_{r,n}-\sigma_{t,n})\\ \Delta_{t,n}&=\frac{\epsilon_{p,n}}{2\sigma_{e,n}}\,(2\sigma_{t,n}-\sigma_{r,n})\end{aligned}\right\}\qquad(20)$$

A typical uniaxial stress-strain curve illustrating the relation between effective stress $\sigma_{e,n}$ and effective plastic strain $\epsilon_{p,n}$ on such a curve is shown in figure 1. Investigations at the National Physical Laboratory of Great Britain on turbine-disk alloys and experiments by Taylor and Quinney (reference 5) were found to correlate well with equations (20) for the range of strains over which the volume of the material is approximately constant.

Equations (20) give the relations between plastic strains and true stresses that will be used as the basis for numerical calculations in the present report. The method of stress analysis to be presented does not depend, however, on the validity of these equations. As more accurate relations are determined between stresses and strains, these relations may readily be used in place of equations (20).

**Calculation of plastic flow when no previous plastic flow has occurred.**—The determination of the plastic stresses in the disk resolves itself into the problem of finding corresponding stresses and strains that satisfy equilibrium and compatibility equations (9) and (10), and biaxial stress-strain equations (20). The problem is approached by first computing the elastic stresses, and the equivalent uniaxial tensile stress at each station is determined from equation (18). If at any station this stress exceeds the proportional elastic limit of the material at the temperature at this station, then plastic flow takes place, and it becomes necessary to resort to a trial-and-error procedure to adjust the stresses to allow for this flow.

Assume, for example, the equivalent uniaxial stress at a given station lies at point A on the extension of the modulus line in figure 2. Because the point A lies above the proportional elastic limit (point B), plastic flow must occur. The stress and the strain must be adjusted to fall on the curved stress-strain curve that is characteristic of the material. As a starting point, the total strain in the true stress-strain condition is assumed equal to the strain at A. The stress-strain condition at the given station then lies on the constant-strain line through B, or at C. The plastic strain $\epsilon_{p,n}$ is

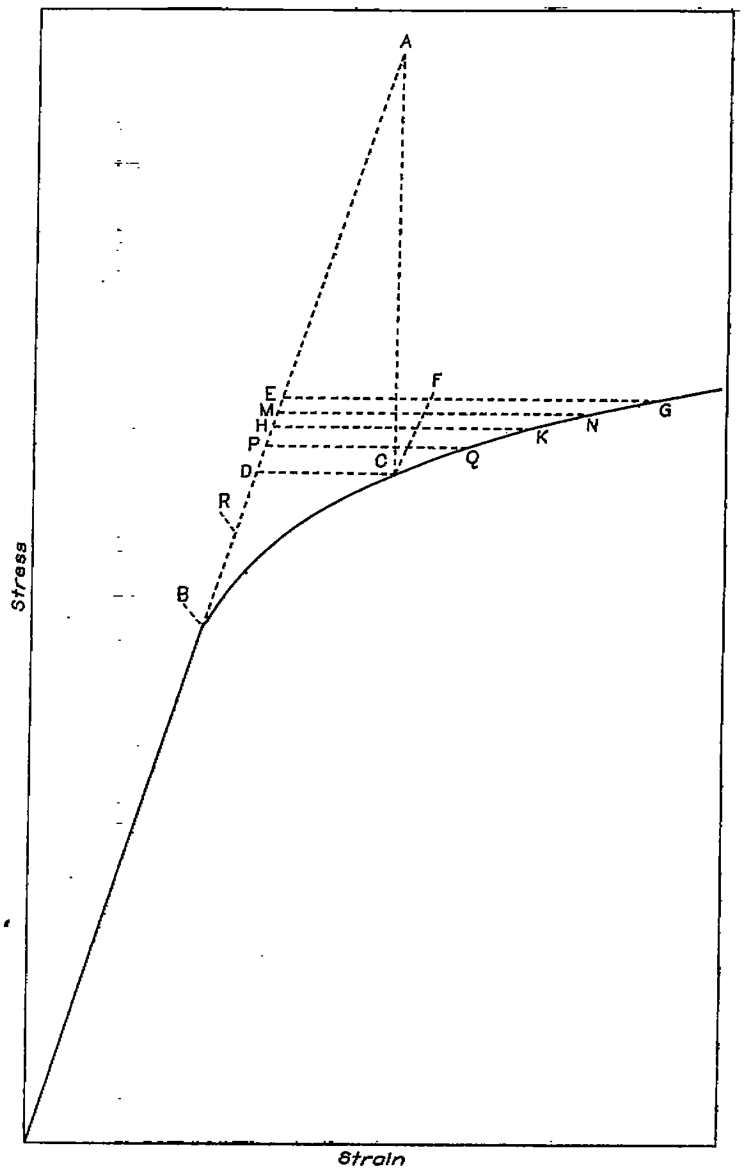



FIGURE 2.—Uniaxial stress-strain curve illustrating procedure used to find correct value of plastic strain.

given by CD. Values of $\Delta_{r,n}$, $\Delta_{t,n}$, and $P'_n$ may be obtained by using this value of $\epsilon_{p,n}$, together with the values of $\sigma_{r,n}$, $\sigma_{t,n}$, and $\sigma_{e,n}$ from the elastic calculations.

Once $P'_n$ has been calculated, new values of $\sigma_{r,n}$, $\sigma_{t,n}$, and $\sigma_{e,n}$ can be computed. The new value of $\sigma_{e,n}$ is greater than that at point D, such as that at point E. Although the stresses corresponding to $\sigma_{e,n}$ at point E together with the strain CD meet the conditions of equations (9) and (10), they locate the stress-strain point F, which is not on the stress-strain curve, so that the physical conditions imposed by the material are as yet unsatisfied. Inasmuch as any value of $\epsilon_{p,n}$ less than CD would give a value of $\sigma_{e,n}$ greater than that at E, CD is a lower limit of $\epsilon_{p,n}$. Similarly, because the value of $\sigma_{e,n}$ calculated by using an $\epsilon_{p,n}$ of CD is too great, the increment of strain EG corresponding to this $\sigma_{e,n}$ is an upper limit of $\epsilon_{p,n}$. Inasmuch as the true value of $\epsilon_{p,n}$ lies between CD and EG, their numerical average, shown as HK, is assumed to be a good approximation. New values of $P'_n$, $\sigma_{r,n}$, $\sigma_{t,n}$, and $\sigma_{e,n}$ can be computed by using HK for $\epsilon_{p,n}$, the stress at E for $\sigma_{e,n}$, and $\sigma_{r,n}$ and $\sigma_{t,n}$. Assume that this new value of $\sigma_{e,n}$ lies at the point M. Because the stress at M is higher than the stress at H in value, the increment HK is too small a value of $\epsilon_{p,n}$ and is therefore established as a new lower limit of $\epsilon_{p,n}$. Further, because M is less than E, the corresponding increment MN is a new upper limit for $\epsilon_{p,n}$ and the process could be repeated again with the numerical average of MN and HK. Similarly, if the calculation using an $\epsilon_{p,n}$ of HK had resulted in a $\sigma_{e,n}$ at P, HK would constitute a new upper limit and PQ a new lower limit. Had the resulting $\sigma_{e,n}$ been at R, HK would have still become the new upper limit of $\epsilon_{p,n}$, but CD would have remained as the lower limit. The process is repeated until the value of $\epsilon_{p,n}$ used in the computation and the $\epsilon_{p,n}$ corresponding to the resulting $\sigma_{e,n}$ are equal.

**Calculation of plastic flow when previous plastic flow has occurred.**—The equations for strain that would apply to a disk that had already undergone the plastic strain are

$$\epsilon_r=\frac{\sigma_r-\mu\sigma_t}{E}+\alpha\Delta T+[\Delta_r]+\Delta_r \tag{21}$$

$$\epsilon_t=\frac{\sigma_t-\mu\sigma_r}{E}+\alpha\Delta T+[\Delta_t]+\Delta_t \tag{21a}$$

Here the terms $[\Delta_r]$ and $[\Delta_t]$ represent strains already existent in the material before the application of stresses $\sigma_t$ and $\sigma_r$ and are constant for the calculation, whereas $\Delta_r$ and $\Delta_t$ represent the components of plastic strain resulting from the application of $\sigma_r$ and $\sigma_t$. In the solution of the equations by the finite-difference method, a term $[P'_n]$ appears together with term $P'_n$. When previous plastic flow has occurred only once, $[P'_n]$ is identical with $P'_n$ from the previous calculation; where plastic flow has previously occurred more than once, $[P'_n]$ is the algebraic sum of all earlier $P'_n$ terms. Thus, the previous plastic flow given by $[P'_n]$ may be grouped with the temperature-effect term $H'_n$ by replacing $H'_n$ with $H'_n-[P'_n]$.

This procedure amounts to an assumption that, as the load and the temperature change, the stress position on the new stress-strain curve would be the same as if a test specimen were loaded above the yield point, the load removed, the temperature changed, and a new load applied. This assumption is illustrated by figure 3, in which point A represents a loading at the first temperature condition; the dotted line AB represents the load-removal path; the curve BCD, the stress-strain curve at the new temperature; and point C, the new stress position. The total strain at point C is given by the sum of three strains. The residual strain caused by the first loading is $\epsilon_1$; $\epsilon_2$ is the elastic part of the strain caused by the second loading; and $\epsilon_3$, the plastic strain caused by the second loading.

When the foregoing procedure is applied, the curve BCD must, of course, represent the true stress-strain curve at the new temperature of a material that has already been subjected to the plastic cycle OAB. In general, the new stress-strain curve is different from the stress-strain curve at the

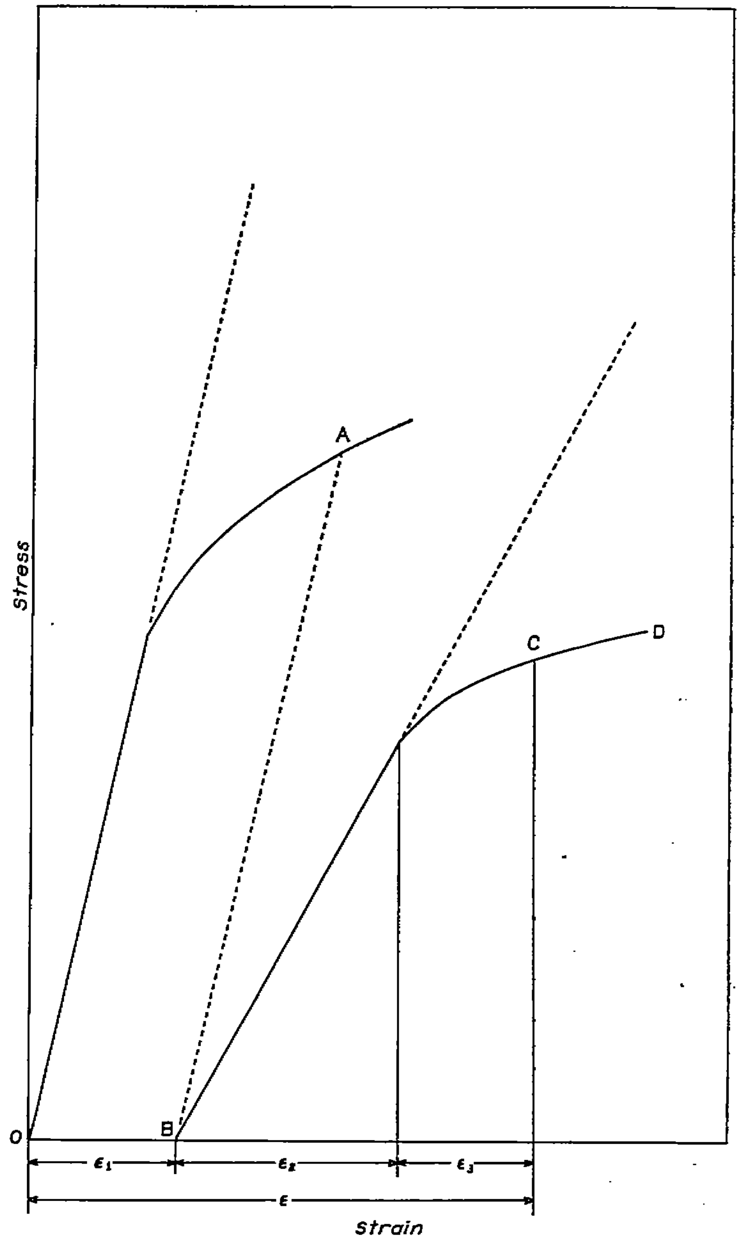



FIGURE 3.—Uniaxial stress-strain curves showing components of strain when plastic flow occurs a second time.

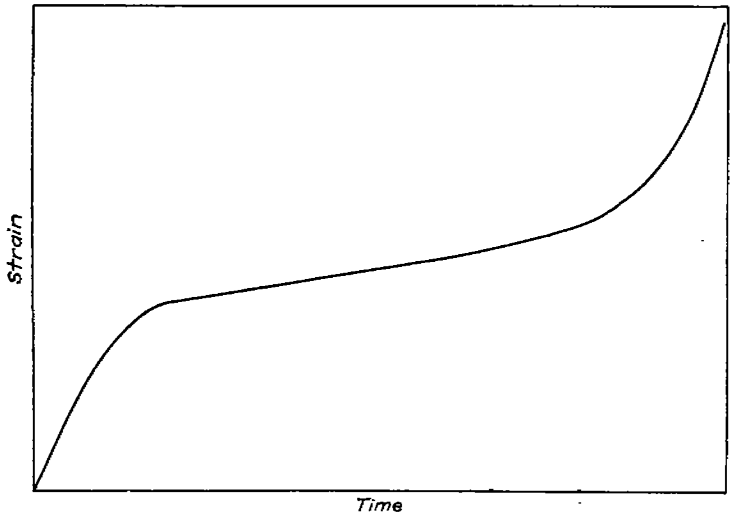



FIGURE 4.—Typical deformation-time curve from a constant-temperature, constant-load creep test.

given temperature of a material that has not been subjected to plastic flow; however, unless data are available it may be necessary to assume that the curve BCD is the stress-strain curve at the given temperature of a specimen of virgin material.

**Calculation of effect of creep.**—Creep is usually defined as the continuous deformation of material under a continuously applied load. Experimental data on creep of various materials are usually obtained from tests run under constant load and temperature, although in many engineering applications of materials the more general problem of changing load and temperature must be considered. The deformation curve obtained in a typical test is shown in figure 4. From this figure it can be seen that the deformation may be considered as having occurred in three stages. During the primary stage, the deformation proceeds at a decreasing rate; during the secondary stage, at a constant rate; and during the tertiary stage, at an increasing rate, which proceeds until failure occurs.

Because of the lack of data on creep except for uniaxial tensile stress, a relation between creep deformation and stress must be assumed. The following equations have been used for calculations in this report but, as better data become available, more accurate relations can be used. By the use of reasoning similar to that employed in determining the biaxial components of plastic-strain formulas for the creep increments, $\delta_{r,n}$ and $\delta_{t,n}$ may be written

$$\delta_{r,n}=\frac{\Gamma_n}{2\sigma_{e,n}}(2\sigma_{r,n}-\sigma_{t,n}) \tag{22}$$

$$\delta_{t,n}=\frac{\Gamma_n}{2\sigma_{e,n}}(2\sigma_{t,n}-\sigma_{r,n}) \tag{22a}$$

In equations (22) and (22a), $\Gamma_n$ represents the total creep that would occur in time $\tau$ under the uniaxial stress $\sigma_{e,n}$. It is here assumed that for sufficiently small values of $\tau$ the creep may be considered as occurring instantaneously at the end of the time period.

During the secondary stage of creep, a characteristic creep rate $c_n$ exists, corresponding to the stress $\sigma_{e,n}$ at temperature $T$, and $\Gamma_n$ is given directly by

$$\Gamma_n=c_n\tau \tag{23}$$

This rate is the value usually published in papers on creep and is the rate used for the numerical calculations of this report. During primary and tertiary creep stages, the creep rate is also a function of time, but does not otherwise complicate the computation.

Once values of $\delta_{r,n}$ and $\delta_{t,n}$ have been found, the values of the $Q'_n$ terms may be determined and new values of $\sigma_{r,n}$ and $\sigma_{t,n}$ may be computed. If the computed values of $\sigma_{r,n}$ and $\sigma_{t,n}$ differ by more than a small amount, perhaps 2 percent, from the values of these stresses before creep occurred, a shorter time interval should be selected and additional computations made for each such time interval required to equal the total time during which creep occurs. The effect of creep that occurred at previous time intervals is considered in a manner similar to that employed in considering previous plastic flow. The successive values of $Q'_n$ are summed to form a term $[Q'_n]$, which gives the total effect of all previous creep deformation so that the term

$$H'_n-[P'_n]$$

is replaced by

$$H'_n-[P'_n]-[Q'_n]$$

In any calculation of stress distribution subsequent to the occurrence of creep, the creep term $[Q'_n]$ is combined with the term $[P'_n]$ as the cumulative effect of all previous plastic deformation.

Examples showing in detail how successive stages of plastic flow and creep are computed, each stage considering all previous plastic deformation, are given in the following section.

## NUMERICAL EXAMPLES

The numerical examples presented here represent a set of computations during one complete start-run-stop cycle for a typical turbine disk with a continuous rim and welded blades. The assumed profile of the disk is shown in figure 5, together with the locations of the point stations used in the computations. The assumed temperature distributions and corre-

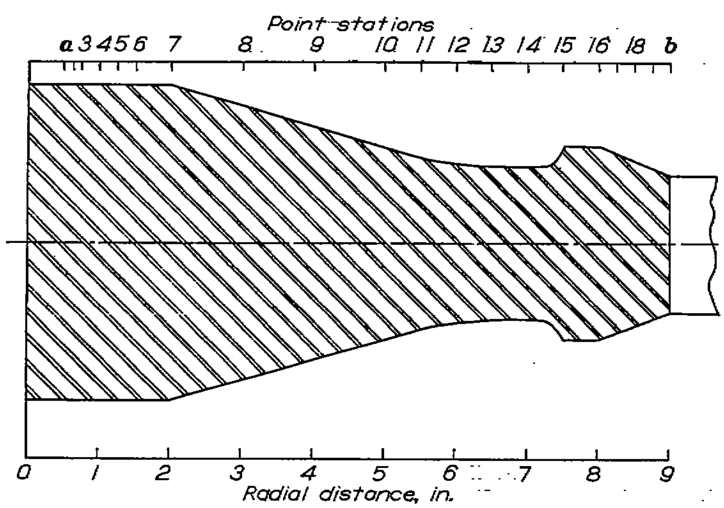



FIGURE 5.—Cross section of disk used for numerical examples showing location of point stations.

sponding turbine rotative speeds are shown in figure 6. Curve IV and the corresponding speed of 11,500 rpm represent the steady-state running condition. Curves I to III and the corresponding speeds represent running conditions through which the turbine disk passes in reaching steady-state operation. Curves V to VII together with the respective speeds represent running conditions through which the turbine disk passes when being stopped. Creep is assumed to occur only during the steady-state running period.

The physical properties of the disk material, including specific gravity, modulus of elasticity, stress-strain characteristics, and thermal coefficients of expansion, were based on the data appearing in reference 6, together with unpublished data obtained from the author of this reference. The stress-strain curves were constructed on the basis of these data and those for example I appear in figure 7. Inasmuch as no data were available on the effect of previous plastic flow on the shape of the stress-strain curves, it was necessary to ignore such effects and to use curves obtained directly from simple tensile-test data. Creep properties corresponding to a material having good creep resistance were assumed. The effect of primary creep was omitted because of lack of data.

Because the disk used for these calculations is solid at the center, a supplementary numerical example showing the computation of the plastic-flow effect on stress distribution in a disk containing a central hole is given in the appendix.

**Example I.**—Example I is the calculation of the stress distribution in a disk operating under the conditions of curve I of figure 6 and having been subjected to no previous plastic deformation. These conditions are assumed to represent disk operation after the first short period of steady combustion when gas temperatures are high, thereby establishing a steep temperature gradient between the center and the rim of the disk.

The preliminary elastic calculation is carried out in table I (a) by the method of reference 1. Two changes are made in the tabular setup. The first change is the insertion of columns 25a and 25b immediately following column 25. Column 25a lists the accumulated values of $[P'_n]$ and $[Q'_n]$, the total effect of previous plastic deformation. For the present example, this column is zero for all stations. Column 25b is the value of the term $H'_n-[P'_n]-[Q'_n]$, which in this example is the same as column 25. The second change is the computation of $M_n$ and $M'_n$ (columns 31 and 32, respectively), which were computed in reference 1 by the use of column 25. In the present computations, column 25b is used. In addition, two more columns, 40 and 41, are added. Column 40 lists the values of the proportional elastic limit $\sigma_{y,n}$ of the material and column 41 lists the values of $\sigma_{e,n}$ as computed from equation (18). The entries in columns 40 and 41 of table I (a) show that the equivalent stress $\sigma_{e,n}$ is less than $\sigma_{y,n}$ for all point stations except 17 to $b$. The effect of plastic flow must be considered at these stations and flow at these stations modifies the stresses at other locations in the disk. With the exceptions and the additions noted, the method of computation is the same as the method of reference 1 and will not be discussed in further detail.

The plastic-flow calculation has been divided into two parts because several quantities used in the computation depend only on the dimensions of the disk and can be used in all subsequent calculations involving plastic deformation. These quantities are computed for stations 17 to $b$, as shown by the four column headings of table I (b).

The second part of the plastic-flow calculation is given in table I (c). The first column in this part of the table (column 46) lists the values of $\epsilon_{p,n}$ obtained from the corresponding stress-strain curve (fig. 7), as previously explained. Column 46a lists the value of $\epsilon_{p,n}$ used for the ensuing calculation, which for the first approximation is the same as column 46.

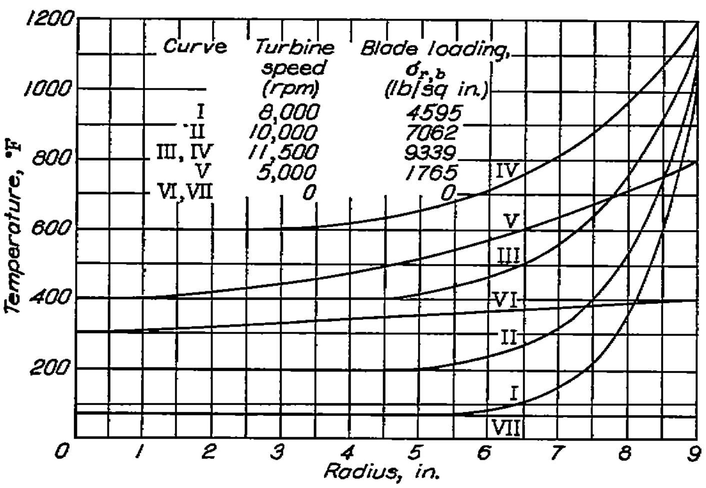
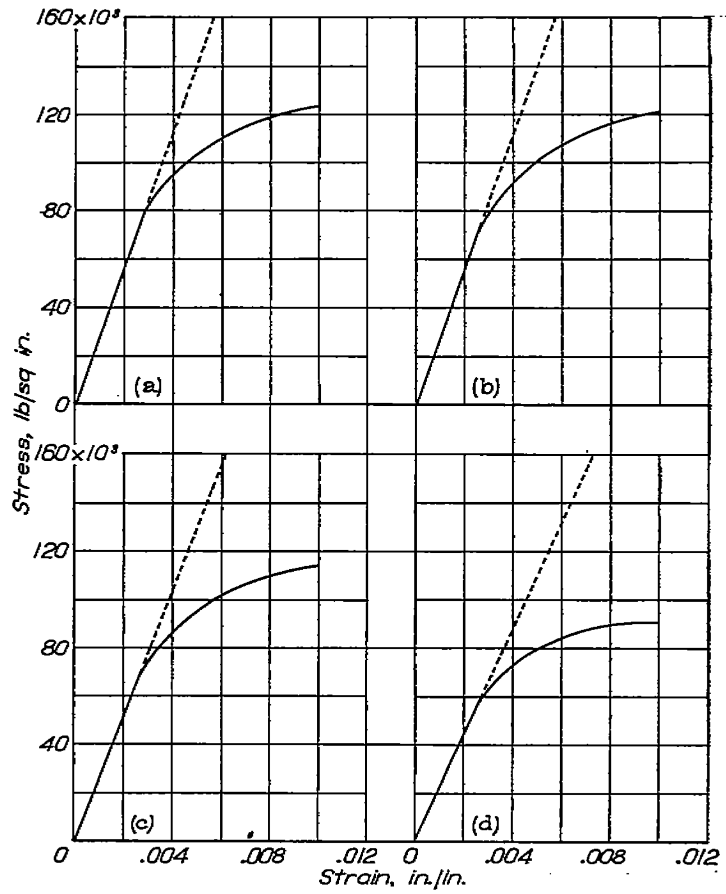



FIGURE 6.—Assumed temperature-distribution curves and corresponding turbine speeds. (Curves I, II, and III are consecutive starting conditions; curve IV represents steady-state operation; and curves V, VI, and VII are consecutive stopping conditions.)




(a) 450° F. (b) 623° F.
(c) 809° F. (d) 1050° F.

FIGURE 7.—Stress-strain curves of disk material for various temperatures.

Columns 47 and 48 list the values of $\Delta_{r,n}$ and $\Delta_{t,n}$, respectively, computed by equations (20). Columns 49 to 52 are computed as shown by the column headings and from these columns the values of $P'_n$ are computed and listed in column 53. Column 54 gives the values of the term $H'_n-[P'_n]-[Q'_n]-P'_n$, which is then used to compute new values of $M_n$, $M'_n$, $B_{r,n}$, $B_{t,n}$, $\sigma_{t,a}$, $\sigma_{r,n}$, $\sigma_{t,n}$, and $\sigma_{e,n}$ as shown in columns 55 to 62, respectively. The new values of $\epsilon_{p,n}$ corresponding to the new values of $\sigma_{e,n}$ are read from figure 7 and listed in column 46 of the second-approximation calculation. The values in column 46 for the first and second approximations now constitute the lower and upper limits, respectively, of the possible strain increments. For the second approximation, column 46a therefore lists as the values of $\epsilon_{p,n}$ to be used in this set of calculations the numerical averages of the two sets of readings from the stress-strain curve. From this value, another new set of stress values is computed and a third set of readings listed in column 46.

At this point in the calculation, two alternate procedures are possible, as shown by consideration of station *b*. Inasmuch as the average value of $4300\times10^{-6}$ inches per inch used in the second approximation gave a graph reading of $1900\times10^{-6}$ inches per inch, the averaging procedure would indicate that the next trial should be $\frac{4300+3960}{2}\times10^{-6}$ or $4130\times10^{-6}$ inches per inch. This value could be used and the procedure continued until the correct value is found. Considerable time may be saved, however, in making the calculation if a weighted approximation is used. Because the plastic-strain value of $3960\times10^{-6}$ inches per inch gave a resulting reading of $4650\times10^{-6}$ inches per inch whereas the value $4300\times10^{-6}$ inches per inch gave the reading $1900\times10^{-6}$ inches per inch, the strain $3960\times10^{-6}$ inches per inch is apparently more nearly correct than $4300\times10^{-6}$ inches per inch. In addition, the shape of the stress-strain curve in the region of $3960\times10^{-6}$ is such that small increases in stress correspond to large changes in strain. If a trial calculation were

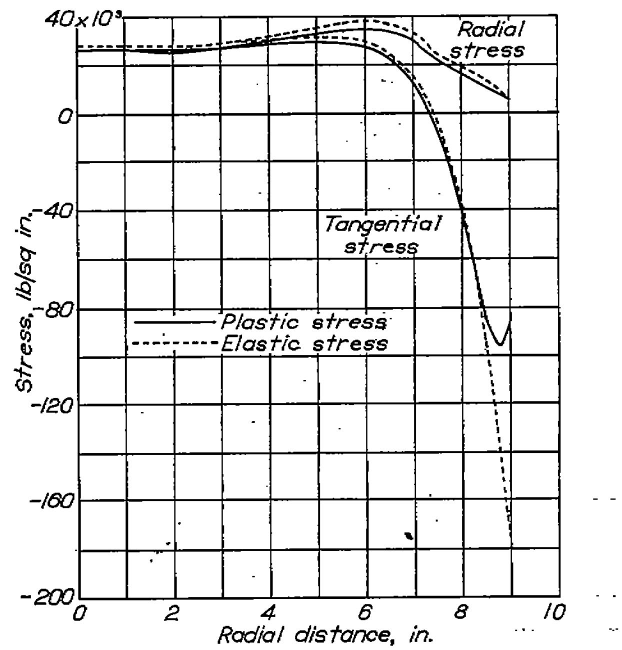



FIGURE 8.—Stresses in turbine disk under conditions of curve I of figure 6.

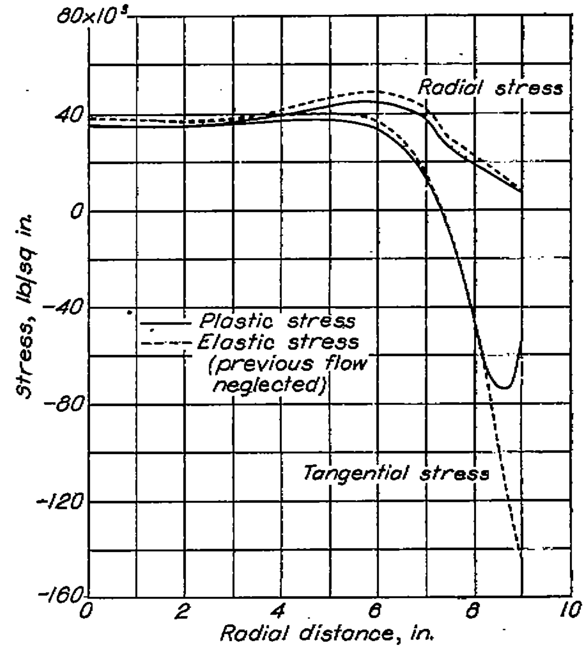



FIGURE 9.—Stresses in turbine disk under conditions of curve II of figure 6.

made using a value closer to 3960 than 4130 (for instance, 4000), more information might be obtained than would be obtained by the averaging procedure. The right answer is thereby more quickly obtained. The same reasoning might be applied to the selection of values to be used at the other stations for the third calculation. The second of these two procedures has been used in table I (c), as can be seen from the values of $\epsilon_{p,n}$ in column 46a used for the third approximation.

Completion of the third approximation and comparison with new values of strain obtained from the stress-strain curve show the estimates of the third approximation to be nearly correct, so that small adjustments made to compute the fourth and fifth approximations give the final answers. A calculation equivalent to a sixth approximation is then made to column 53 to get the final correct values of the $P'_n$ terms. The stresses at the other stations *a* through 16 can now be computed by using the value of $\sigma_{t,a}$ found in the sixth approximation with the values $A_{r,n}$, $A_{t,n}$, $B_{r,n}$, and $B_{t,n}$ found in table I (a). The values of plastic stress at all radii together with the elastic-stress distribution are plotted in figure 8.

**Example II.**—Example II considers the disk studied in example I at the time that the operating conditions have reached those indicated by curve II of figure 6. The elastic calculations are made by the method of reference 1 modified in accordance with the changes made in example I. The essential parts of the computation are shown in table II, which is abridged from the complete calculation. Column 25a lists the values of $[P'_n]$ that were found as the final values of $P'_n$ in example I. Plastic flow occurs at stations 17, 18, 19, and *b*, and calculated true stresses when this plastic flow is considered are listed in table II. The stresses obtained as a result of this computation are plotted in figure 9. The elastic stresses obtained without considering the plastic flow that occurred previously are also plotted for comparison in figure 9.

**Example III.**—Example III continues the cycle analyzed in examples I and II, at the conditions of curve III of figure 6.

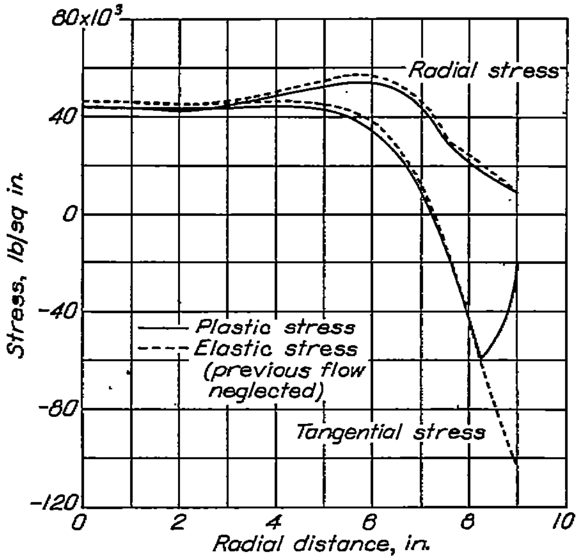



FIGURE 10.—Stresses in turbine disk under conditions of curve III of figure 6.

Table III gives the essential parts of the calculation for this example, which is similar to the procedure for example I in table I. The value of $[P'_n]$, column 25a, however, is the total of the values of $P'_n$ obtained from examples I and II. In this example, plastic flow occurs only at stations 17 and 18. The results of this computation, together with the elastic-stress curves found without consideration of previous plastic flow, are shown in figure 10.

**Example IV.**—The steady-state operating conditions represented by curve IV of figure 6 are treated in example IV. The essential calculations shown in table IV (a) were made similarly to those in table III except that $[P'_n]$ in column 25a is now the sum of the values of $P'_n$ from examples I, II, and III. Because no values of $\sigma_{e,n}$ exceed those of $\sigma_{y,n}$, no plastic flow occurs and the stress values of table IV (a) are the true stresses at the beginning of steady-state operation. However, as parts of the disk are at elevated temperature, significant creep can occur at steady load at stations 16, 17, and 18 where the stresses are sufficiently high. Table IV (b) shows the calculations of creep. Column 63 lists the creep rate $c_n$ (in./(in.)(hr)), and column 64 the creep increment $\Gamma_n$ for the 5-hour running period. Columns 65 and 66 give the computed values of $\delta_{r,n}$ and $\delta_{t,n}$, respectively. The computation

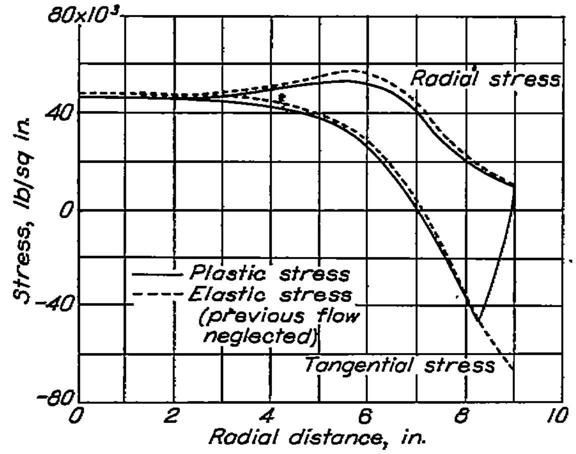



FIGURE 11.—Stresses in turbine disk under conditions of curve IV of figure 6. (The stresses before and after creep occurs coincide within the accuracy of this plot.)

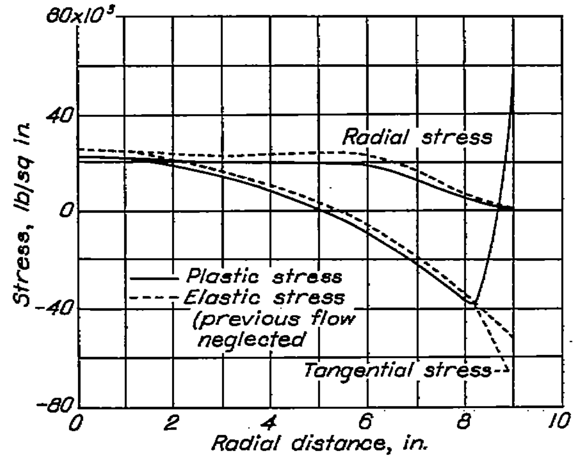



FIGURE 12.—Stresses in turbine disk under conditions of curve V of figure 6.

then proceeds in a manner similar to the plastic-flow calculations, as indicated in the column headings. The values for stress obtained indicate that, for small values, creep has only a slight effect on the stresses. Figure 11 shows the stress distributions at the beginning and the end of the steady-state running period, together with the elastic stresses obtained without considering either creep or previous plastic flow.

**Example V.**—The conditions of example V represent one of the conditions through which the turbine disk is assumed to pass during the stopping period. The abridged elastic calculations are given in table V. Values listed now represent the accumulated effect of plastic flow $[P'_n]$ plus the additional effect of the creep represented by $[Q'_n]$; $[Q'_n]$ is the same as the $Q'_n$ computed in example IV. All values of $\sigma_{e,n}$ are less than the corresponding values of $\sigma_{y,n}$; therefore no plastic flow occurs. The results of the calculation are plotted in figure 12, together with the elastic stresses computed without considering previous plastic flow or creep.

**Example VI.**—Example VI is the computation of the stress distribution at the temperature distribution assumed to be present shortly after the wheel has stopped turning. The essential parts of the calculation are shown in table VI. Because no flow was found in example V, the $[P'_n]+[Q'_n]$ term will be the same in this example as in example V. Plastic flow occurs at station *b*. The resulting stresses are plotted in figure 13, together with the elastic stresses com-

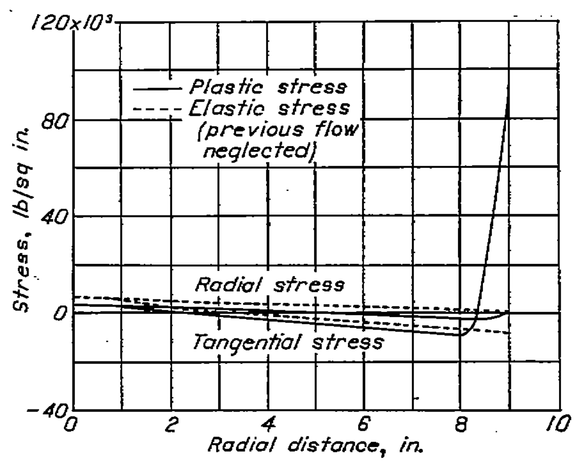



FIGURE 13.—Stresses in turbine disk under conditions of curve VI of figure 6.

puted without consideration of previous plastic flow and creep.

**Example VII.**—Example VII is the computation of the stress distribution in the disk after the temperature has become uniform at the ambient temperature (assumed to be 70° F) throughout the disk. These stresses are therefore the residual stresses in the disk resulting from the flow occurring during the complete operating cycle. The abridged calculations given in table VII indicate that plastic flow occurs at station *b* and the residual stresses are plotted in figure 14.

**Discussion of numerical examples.**—The foregoing cycle of stress calculations is indicative of the means of obtaining a complete analysis of the stress behavior of a turbine disk. Although the results plotted in the various figures and summarized in figure 15 do not represent the exact behavior of any particular turbine disk because of the lack of data on the material properties and temperature gradients, they do give a qualitative picture of the behavior of a turbine disk with welded blades. The high residual tensile stress at the rim of the wheel provides a plausible explanation of the rim cracking that has occurred in such wheels. The compressive flow at the rim during starting and the tensile flow on stopping result in cyclic flow of the rim material with each start and stop and possibly induce cracks. When accurate data are available on creep, stress-strain relations, the effect of strain-hardening, and temperature distribution, quantitative

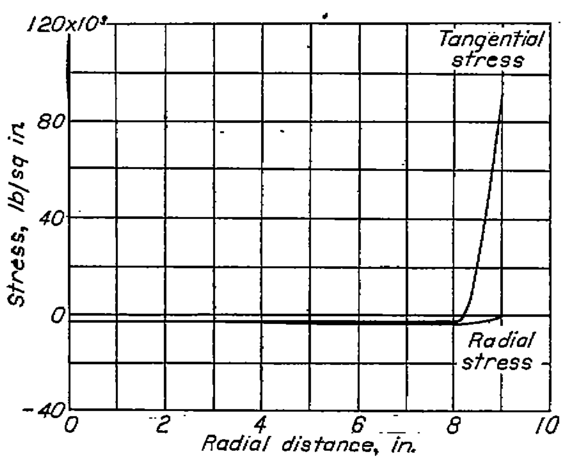



FIGURE 14.—Residual stresses in turbine disk upon completion of one operating cycle.

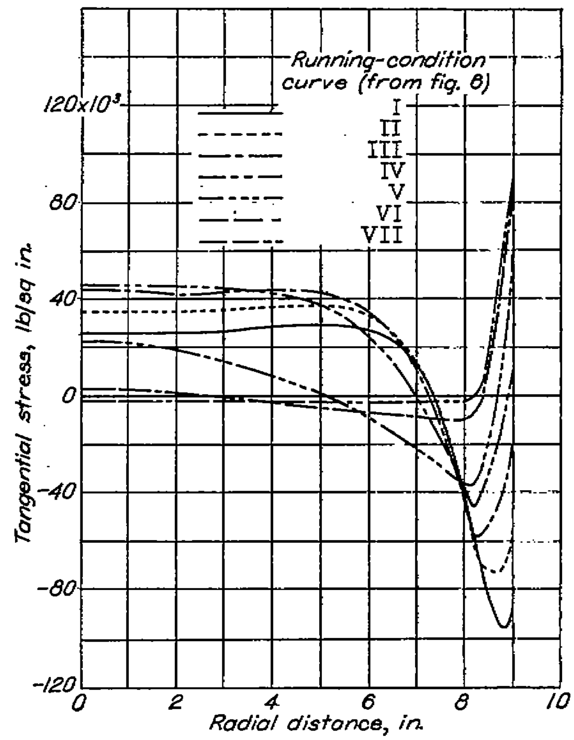



FIGURE 15.—Plastic tangential stresses in turbine disk during one running cycle.

analyses of disk behavior will be available as a guide in future turbine design.

## CONCLUSIONS

A method for studying the operating stresses in gas-turbine disks has been presented that includes consideration of the effect of plastic flow and creep on the stress distribution. Results of calculations indicate that rim cracking in turbine wheels with welded blade attachments may be caused by alternate compressive and tensile plastic flow as the wheel is alternately heated and cooled. From the results of the numerical examples presented, it may be concluded that plastic flow markedly alters the elastic-stress distribution.

Flight Propulsion Research Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, *March 5, 1948.*

# APPENDIX

## STRESS CALCULATION FOR DISK WITH CENTRAL HOLE

The calculations given in the section Numerical Examples deal with a disk that is solid at the center and has temperature gradients such that the plastic flow is confined to the region of the rim. Disks of other types spun under different conditions may be subject to plastic flow in other regions.

One example of such a disk is a parallel-sided disk with a central hole spun at a uniform temperature. In this disk plastic flow first occurs in the region of the central hole. Such a disk spun at a speed great enough to cause some flow near the hole is calculated here.

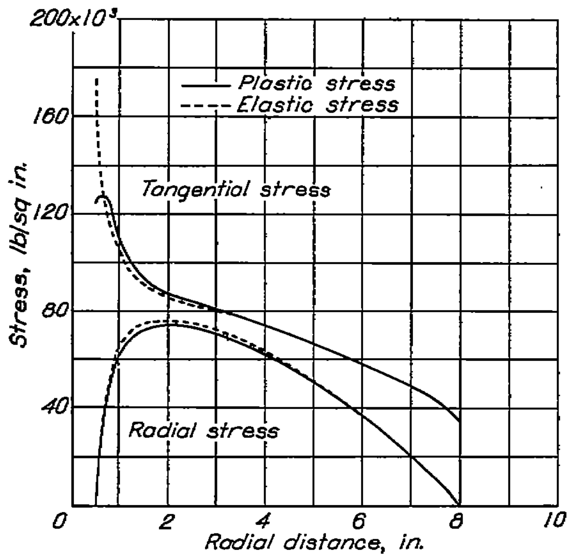



FIGURE 16.—Stresses in parallel-sided disk with central hole.

The essential columns of the elastic calculation are given in table VIII (a). Flow is indicated at stations $a$, 2, 3, and 4. However, as flow occurs the stresses farther out in the disk may be increased. The quantities depending on disk dimensions, together with the first approximation, shown in table VIII (b), are found in the manner given in the text. When the values of $B_{r,n}$ and $B_{t,n}$ (columns 57 and 58) are found for the stations at which flow occurs, however, new values of $B_{r,n}$ and $B_{t,n}$ must also be computed for all other point stations before a new value of $\sigma_{t,a}$ (column 59) can be found. These computations are also shown in table VIII (b) for the first approximation. Additional approximations must be made in the same manner until the correct flow increments are found. The stresses so calculated are plotted in figure 16.

Where large numbers of computations involving plastic flow at the center of the disk are to be made, it may be desirable to change the finite-difference approach to the problem in such a manner that the calculations are made from the outside of the disk toward the center instead of from the center toward the rim. This procedure has certain disadvantages as a general approach to the problem of stresses in disks, particularly in that it requires a greater number of significant figures to obtain the same accuracy. For special applications it may, however, present advantages that outweigh the disadvantages in more general problems.

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## TABLE I.—CALCULATION OF STRESS DISTRIBUTION FOR EXAMPLE I

(a) Elastic-stress calculation

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $r_n$ | $h_n$ | $\rho_n\omega^2$ | $\mu_n$ | $E_n$ | $\alpha_n$ | $\Delta T_n$ | $C_n$, (1)×(2) | $\frac{(1)-(1)_{n-1}}{2}$ | $D_n$, (2)×(9) | $G_n$, $(2)_{n-1}$×(9) | (3)×(8)×(1) | (12)+$(12)_{n-1}$ | $H_n$, (9)×(13) | 1/(5) |
| a | 0.5000 | 4.3750 | Constant at 534.78 | Constant at 0.35000 | 30.400×10⁶ | 8.2970×10⁻⁶ | 0 | 2.1875 | ------- | ------- | ------- | 584.86 | --------- | --------- | 0.032895×10⁻⁶ |
| 2 | .6250 | 4.3750 | | | 30.400 | 8.2970 | 0 | 2.7344 | 0.06250 | 0.27344 | 0.27344 | 913.85 | 1,498.7 | 93.672 | .032895 |
| 3 | .7500 | 4.3750 | | | 30.400 | 8.2970 | 0 | 3.2812 | .06250 | .27344 | .27344 | 1,315.9 | 2,229.8 | 139.36 | .032895 |
| 4 | 1.0000 | 4.3750 | | | 30.400 | 8.2970 | 0 | 4.3750 | .12500 | .54688 | .54688 | 2,339.4 | 3,655.3 | 456.92 | .032895 |
| 5 | 1.2500 | 4.3750 | | | 30.400 | 8.2970 | 0 | 5.4688 | .12500 | .54688 | .54688 | 3,655.4 | 5,994.8 | 749.37 | .032895 |
| 6 | 1.5000 | 4.3750 | | | 30.400 | 8.2970 | 0 | 6.5625 | .12500 | .54688 | .54688 | 5,263.8 | 8,919.2 | 1,114.9 | .032895 |
| 7 | 2.0000 | 4.3750 | | | 30.400 | 8.2970 | 0 | 8.7500 | .25000 | 1.0938 | 1.0938 | 9,357.8 | 14,622 | 3,655.4 | .032895 |
| 8 | 3.0000 | 3.8400 | | | 30.400 | 8.2970 | 0 | 11.520 | .50000 | 1.9200 | 2.1875 | 18,480 | 27,838 | 13,919 | .032895 |
| 9 | 4.0000 | 3.2750 | | | 30.400 | 8.2970 | 0 | 13.100 | .50000 | 1.6375 | 1.9200 | 28,020 | 46,500 | 23,250 | .032895 |
| 10 | 5.0000 | 2.6800 | | | 30.300 | 8.3020 | 3.00 | 13.400 | .50000 | 1.3400 | 1.6375 | 35,827 | 63,847 | 31,923 | .033003 |
| 11 | 5.5000 | 2.3720 | | | 30.300 | 8.3080 | 7.00 | 13.046 | .25000 | .59300 | .67000 | 38,368 | 74,195 | 18,548 | .033003 |
| 12 | 6.0000 | 2.2100 | | | 30.200 | 8.3240 | 17.00 | 13.260 | .25000 | .55250 | .59300 | 42,543 | 80,911 | 20,228 | .033112 |
| 13 | 6.5000 | 2.1600 | | | 30.100 | 8.3580 | 38.00 | 14.040 | .25000 | .54000 | .55250 | 48,799 | 91,342 | 22,836 | .033222 |
| 14 | 7.0000 | 2.1550 | | | 30.000 | 8.4240 | 79.00 | 15.085 | .25000 | .53875 | .54000 | 56,467 | 105,270 | 26,317 | .033333 |
| 15 | 7.5000 | 2.7000 | | | 29.600 | 8.5500 | 158.00 | 20.250 | .25000 | .67500 | .53875 | 81,212 | 137,680 | 34,420 | .033784 |
| 16 | 8.0000 | 2.7000 | | | 28.900 | 8.7810 | 302.00 | 21.600 | .25000 | .67500 | .67500 | 92,401 | 173,620 | 43,404 | .034602 |
| 17 | 8.2500 | 2.5500 | | | 28.400 | 8.9550 | 410.00 | 21.038 | .12500 | .31875 | .33750 | 92,808 | 185,210 | 23,151 | .035211 |
| 18 | 8.5000 | 2.3800 | | | 27.500 | 9.1840 | 553.00 | 20.230 | .12500 | .29750 | .31875 | 91,952 | 184,760 | 23,095 | .036364 |
| 19 | 8.7500 | 2.1450 | | | 26.000 | 9.4820 | 739.00 | 18.769 | .12500 | .26812 | .29750 | 87,819 | 179,770 | 22,471 | .038462 |
| b | 9.0000 | 1.9100 | | | 22.100 | 9.8690 | 980.00 | 17.190 | .12500 | .23875 | .26812 | 82,728 | 170,550 | 21,319 | .045249 |

| $n$ | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 25a |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | (4)×(15) | $\frac{[1+(4)]\times(15)}{(1)}$ | (17)×(9) | $(17)_{n-1}$×(9) | $C'_n$, (16)+(18) | $D'_n$, (15)+(18) | $F'_n$, $(16)_{n-1}$−(19) | $G'_n$, $(15)_{n-1}$−(19) | (6)×(7) | $H'_n$, (24)−$(24)_{n-1}$ | $[P'_n]+[Q'_n]$ |
| a | 0.011513×10⁻⁶ | 0.088816×10⁻⁶ | ---------- | ---------- | ---------- | ---------- | ---------- | ---------- | 0 ×10⁻⁶ | ---------- | ------ |
| 2 | .011513 | .071053 | 0.0044410×10⁻⁶ | 0.0055510×10⁻⁶ | 0.015954×10⁻⁶ | 0.037336×10⁻⁶ | 0.0059620×10⁻⁶ | 0.027344×10⁻⁶ | 0 | 0 ×10⁻⁶ | Constant at 0 |
| 3 | .011513 | .059211 | .0037010 | .0044410 | .015214 | .036596 | .0070720 | .028454 | 0 | 0 | |
| 4 | .011513 | .044408 | .0055510 | .0074010 | .017064 | .038446 | .0041120 | .025494 | 0 | 0 | |
| 5 | .011513 | .035526 | .0044410 | .0055510 | .015954 | .037336 | .0059620 | .027344 | 0 | 0 | |
| 6 | .011513 | .029605 | .0037010 | .0044410 | .015214 | .03[illegible]96 | .0070720 | .028454 | 0 | 0 | |
| 7 | .011513 | .022204 | .0055510 | .0074010 | .017064 | .038446 | .0041120 | .025494 | 0 | 0 | |
| 8 | .011513 | .014803 | .0074010 | .011102 | .018914 | .040296 | .00041100 | .021793 | 0 | 0 | |
| 9 | .011513 | .011102 | .0055510 | .0074010 | .017064 | .038446 | .0041120 | .025494 | 0 | 0 | |
| 10 | .011551 | .0089108 | .0044550 | .0055510 | .016006 | .037458 | .0059620 | .027344 | 24.906 | 24.906 | |
| 11 | .011551 | .0081007 | .0020250 | .0022280 | .013576 | .035028 | .0093230 | .030775 | 58.156 | 33.250 | |
| 12 | .011589 | .0074502 | .0018030 | .0020250 | .013452 | .034975 | .0095260 | .030978 | 141.508 | 83.352 | |
| 13 | .011628 | .0069000 | .0017250 | .0018630 | .013358 | .034947 | .0097260 | .031249 | 317.604 | 176.10 | |
| 14 | .011667 | .0064286 | .0016070 | .0017250 | .013274 | .034940 | .0099030 | .031497 | 665.496 | 347.89 | |
| 15 | .011824 | .0060611 | .0015200 | .0016070 | .013344 | .035304 | .010060 | .031726 | 1350.90 | 685.40 | |
| 16 | .012111 | .0058391 | .0014600 | .0015200 | .013571 | .036062 | .010304 | .032204 | 2651.80 | 1301.0 | |
| 17 | .012324 | .0057618 | .0007200 | .00073000 | .013044 | .035931 | .011381 | .033872 | 3671.55 | 1019.7 | |
| 18 | .012727 | .0057754 | .00072200 | .00072000 | .013449 | .037086 | .011604 | .034491 | 5078.75 | 1407.2 | |
| 19 | .013462 | .0059342 | .00074200 | .00072200 | .014204 | .039204 | .012005 | .035642 | 7007.20 | 1928.4 | |
| b | .015837 | .0067873 | .00084800 | .00074200 | .016685 | .046097 | .012720 | .037720 | 9671.62 | 2064.4 | |

| $n$ | 25b | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $H'_n-[P'_n]-[Q'_n]$, (25)−(25a) | (20)×(10)−(8)×(21) | $K_n$, [(22)×(10)−$(8)_{n-1}$×(21)]+(26) | $L_n$, [(23)×(10)+(11)×(21)]+−(26) | $K'_n$, [(8)×(22)−(20)×$(8)_{n-1}$]+(26) | $L'_n$, [(20)×(11)+(8)×(23)]÷−(26) | $M_n$, [(25b)×(10)+(14)×(21)]+(26) | $M'_n$, [(20)×(14)+(8)×(25b)]+(26) | $A_{r,n}$, (27)×$(33)_{n-1}$+(28)×$(34)_{n-1}$ | $A_{t,n}$, (29)×$(33)_{n-1}$+(30)×$(34)_{n-1}$ | $B_{t,n}$, (27)×$(35)_{n-1}$+(28)×$(36)_{n-1}$+(31) |
| a | ---------- | ---------- | ------ | ------ | ------ | ------ | ------ | ------ | 1.0000 | 1.0000 | 0 |
| 2 | 0 ×10⁻⁶ | −0.097729×10⁻⁶ | 0.81902 | 0.18097 | 0.19029 | 0.80971 | −35.786 | −15.292 | .99999 | 1.0000 | −35.786 |
| 3 | 0 | −.11592 | .84657 | .15344 | .15870 | .84130 | −43.996 | −18.290 | 1.0000 | 1.0000 | −76.038 |
| 4 | 0 | −.15887 | .77988 | .22010 | .23919 | .76080 | −110.57 | −49.077 | .99998 | .99909 | −178.45 |
| 5 | 0 | −.19546 | .81001 | .18097 | .19029 | .80970 | −143.14 | −61.166 | .99996 | .99908 | −306.50 |
| 6 | 0 | −.23184 | .84657 | .15344 | .15870 | .84131 | −175.99 | −78.163 | .99997 | .99909 | −401.97 |
| 7 | 0 | −.31774 | .77990 | .22011 | .23920 | .76080 | −442.30 | −196.81 | .99998 | .99909 | −801.33 |
| 8 | 0 | −.42790 | .82216 | .30379 | .37570 | .68341 | −1310.8 | −615.25 | 1.1259 | 1.0591 | −2,173.8 |
| 9 | 0 | −.47570 | .91659 | .24298 | .30000 | .77094 | −1879.1 | −834.01 | 1.2806 | 1.1542 | −4,185.0 |
| 10 | 24.906 | −.48049 | 1.0046 | .20391 | .27012 | .81712 | −2558.1 | −1,758.0 | 1.5309 | 1.2915 | −7,267.8 |
| 11 | 83.250 | −.44892 | 1.0333 | .092930 | .13430 | .91461 | −1491.2 | −1,527.2 | 1.7019 | 1.3865 | −9,457.7 |
| 12 | 83.352 | −.45634 | .98834 | .082955 | .10777 | .91762 | −1651.2 | −3,018.3 | 1.7971 | 1.4560 | −11,579 |
| 13 | 176.10 | −.48345 | .94706 | .074844 | .083789 | .92277 | −1847.4 | −5,744.9 | 1.8120 | 1.4941 | −13,503 |
| 14 | 347.89 | −.51992 | .93326 | .068927 | .071127 | .92764 | −2129.1 | −10,766 | 1.7041 | 1.5149 | −15,952 |
| 15 | 685.40 | −.70590 | .74482 | .057282 | −.0034293 | .92030 | −2376.8 | −20,313 | 1.4231 | 1.3880 | −15,800 |
| 16 | 1301.0 | −.76978 | .93962 | .059913 | .067872 | .91728 | −3174.2 | −27,271 | 1.4208 | 1.3697 | −20,718 |
| 17 | 1019.7 | −.75176 | 1.0276 | .030493 | .056290 | .95376 | −1538.9 | −28,938 | 1.5013 | 1.3303 | −25,257 |
| 18 | 1407.2 | −.74625 | 1.0409 | .029591 | .064578 | .94076 | −1706.7 | −38,564 | 1.6037 | 1.4011 | −31,137 |
| 19 | 1928.4 | −.73201 | 1.0791 | .028988 | .084733 | .91965 | −1909.8 | −49,881 | 1.7712 | 1.4244 | −39,567 |
| b | 2064.4 | −.78842 | 1.0935 | .027099 | .11986 | .82809 | −2053.3 | −58,543 | 1.9754 | 1.3918 | −50,232 |

## TABLE I.—CALCULATION OF STRESS DISTRIBUTION FOR EXAMPLE I—Concluded

(a) Elastic-stress calculation—Concluded

| $n$ | 36 | 37 | 38 | 39 | 40 | 41 |
|---|---|---|---|---|---|---|
| | $B_{t,n}$, $(29)\times(35)_{n-1}+$ $(30)\times(36)_{n-1}+$ $(32)$ | $\sigma_{t,a}$, $[\sigma_{r,b}-(35)_b]$ $+(33)_b$ | $\sigma_{r,n}$, $(33)\times(37)+$ $(35)$ | $\sigma_{t,n}$, $(34)\times(37)$ $+(36)$ | $\sigma_{y,n}$ | $\sigma_{e,n}$, $\sqrt{(38)^2+(39)^2-(38)\times(39)}$ |
| $a$ | 0 | ---------- | 27,755 | 27,755 | 73,500 | 27,755 |
| 2 | −15.292 | | 27,719 | 27,740 | 73,500 | 27,730 |
| 3 | −36.834 | | 27,678 | 27,718 | 73,500 | 27,698 |
| 4 | −95.431 | | 27,576 | 27,659 | 73,500 | 27,618 |
| 5 | −172.39 | | 27,447 | 27,582 | 73,500 | 27,515 |
| 6 | −266.85 | | 27,292 | 27,488 | 73,500 | 27,391 |
| 7 | −509.83 | | 26,893 | 27,245 | 73,500 | 27,071 |
| 8 | −1,287.3 | | 29,076 | 28,108 | 73,500 | 28,604 |
| 9 | −2,478.6 | Constant at 27,755 for $\sigma_{r,b}$ of 4595 lb/sq in. (fig. 6) | 31,608 | 29,559 | 73,500 | 30,635 |
| 10 | −4,913.8 | | 35,222 | 30,932 | 73,500 | 33,285 |
| 11 | −6,997.5 | | 37,779 | 31,493 | 73,500 | 35,061 |
| 12 | −10,459 | | 38,300 | 29,952 | 73,500 | 34,883 |
| 13 | −16,366 | | 36,689 | 25,103 | 73,500 | 32,435 |
| 14 | −26,913 | | 33,843 | 15,131 | 73,500 | 29,364 |
| 15 | −45,028 | | 23,698 | −6,504 | 73,500 | 27,532 |
| 16 | −79,644 | | 18,702 | −41,628 | 73,500 | 53,490 |
| 17 | −106,070 | | 16,412 | −67,593 | 72,000 | 77,120 |
| 18 | −139,980 | | 13,374 | −101,090 | 70,300 | 108,400 |
| 19 | −181,250 | | 9,593 | −141,720 | 67,000 | 146,750 |
| $b$ | −213,380 | | 4,595 | −174,750 | 64,000 | 177,090 |

(b) Constants determined by disk geometry to be used in all plastic calculations

| $n$ | 42 | 43 | 44 | 45 |
|---|---|---|---|---|
| | $(9)/(1)$ | $(9)/(1)_{n-1}$ | $1+(42)$ | $1-(43)$ |
| 17 | 0.015152 | 0.015625 | 1.0152 | 0.98438 |
| 18 | .014706 | .015152 | 1.0147 | .98485 |
| 19 | .014286 | .014706 | 1.0143 | .98529 |
| $b$ | .013889 | .014286 | 1.0139 | .98571 |

(c) Plastic-stress calculation

| Approximation | $n$ | 46 | 46a | 47 | 48 | 49 | 50 | 51 | 52 | 53 |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $\epsilon_{p,n}$ (graph reading, fig. 7) | $\epsilon_{p,n}$ (estimate) | $\Delta_{r,n}$, $\frac{(46a)}{(62)\times 2}\times$ $[2(60)-(61)]$ | $\Delta_{t,n}$, $\frac{(46a)}{(62)\times 2}\times$ $[2(61)-(60)]$ | $(47)\times(42)$ | $(47)_{n-1}\times(43)$ | $(48)\times(44)$ | $(48)_{n-1}\times(45)$ | $P'_n$, $(49)+(50)-$ $(51)+(52)$ |
| 1 | 17 | 20×10⁻⁶ | 20×10⁻⁶ | 13.021×10⁻⁶ | −19.657×10⁻⁶ | 0.19729×10⁻⁶ | 0.00000×10⁻⁶ | −19.956×10⁻⁶ | 0.000×10⁻⁶ | 20.153×10⁻⁶ |
| | 18 | 670 | 670 | 395.07 | −666.15 | 5.8099 | .19729 | −675.94 | −19.359 | 662.59 |
| | 19 | 1850 | 1850 | 1014.2 | −1847.1 | 14.489 | 5.8099 | −1873.5 | −656.35 | 1237.4 |
| | $b$ | 3960 | 3960 | 2056.6 | −3959.1 | 28.564 | 14.489 | −4014.1 | −1820.7 | 2236.5 |
| 2 | 17 | 20 | 20 | 12.562 | −19.759 | .19034 | 0 | −20.059 | 0 | 20.249 |
| | 18 | 690 | 685 | 402.14 | −681.31 | 5.9139 | .19034 | −691.33 | −19.460 | 677.97 |
| | 19 | 1880 | 1865 | 1040.3 | −1860.7 | 14.862 | 5.9139 | −1887.3 | −671.29 | 1236.8 |
| | $b$ | 4650 | 4300 | 2312.1 | −4295.8 | 32.113 | 14.862 | −4355.5 | −1834.1 | 2508.4 |
| 3 | 17 | 20 | 20 | 12.544 | −19.762 | .19007 | 0 | −20.062 | 0 | 20.252 |
| | 18 | 670 | 670 | 393.05 | −666.43 | 5.7802 | .19007 | −676.23 | −19.463 | 662.74 |
| | 19 | 1850 | 1860 | 1036.5 | −1855.8 | 14.807 | 5.7802 | −1882.3 | −656.63 | 1246.3 |
| | $b$ | 1900 | 4100 | 2218.3 | −4095.2 | 30.810 | 14.807 | −4152.1 | −1829.3 | 2368.4 |
| 4 | 17 | 20 | 20 | 12.555 | −19.760 | .19023 | 0 | −20.060 | 0 | 20.250 |
| | 18 | 690 | 680 | 398.99 | −676.36 | 5.8675 | .19023 | −686.30 | −19.461 | 672.90 |
| | 19 | 1960 | 1860 | 1037.2 | −1855.7 | 14.817 | 5.8675 | −1882.2 | −666.42 | 1236.5 |
| | $b$ | 2900 | 4000 | 2156.0 | −3995.8 | 29.945 | 14.817 | −4051.3 | −1829.2 | 2266.9 |
| 5 | 17 | 20 | 20 | 12.558 | −19.759 | .19028 | 0 | −20.059 | 0 | 20.249 |
| | 18 | 680 | 680 | 399.27 | −676.33 | 5.8717 | .19028 | −686.27 | −19.460 | 672.87 |
| | 19 | 1860 | 1860 | 1037.5 | −1855.7 | 14.822 | 5.8717 | −1882.2 | −666.38 | 1236.5 |
| | $b$ | 4200 | 4015 | 2160.3 | −4011.0 | 30.004 | 14.822 | −4066.8 | −1829.2 | 2282.4 |
| 6 | 17 | 20 | 20 | 12.557 | −19.760 | .19026 | 0 | −20.060 | 0 | 20.250 |
| | 18 | 680 | 680 | 399.25 | −676.33 | 5.8714 | .19026 | −686.27 | −19.461 | 672.87 |
| | 19 | 1860 | 1860 | 1037.5 | −1855.7 | 14.822 | 5.8714 | −1882.2 | −666.38 | 1236.5 |
| | $b$ | 4015 | 4015 | 2160.8 | −4011.0 | 30.011 | 14.822 | −4066.8 | −1829.2 | 2282.4 |

| Approximation | $n$ | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $H'_n-[P'_n]-$ $[Q'_n]-P'_n$, $(25b)-(53)$ | $M_n$, $(54)\times(10)+$ $(14)\times(21)+$ $(26)$ | $M'_n$, $(54)\times(8)+$ $(14)\times(20)+$ $(26)$ | $B_{r,n}$, $(27)\times(57)_{n-1}+$ $(28)\times(58)_{n-1}+$ $(55)$ | $B_{t,n}$, $(29)\times(57)_{n-1}+$ $(30)\times(58)_{n-1}+$ $(56)$ | $\sigma_{t,a}$, $[\sigma_{r,b}-(57)_b]$ $+(33)_b$ | $\sigma_{r,n}$, $(33)\times(59)+$ $(57)$ | $\sigma_{t,n}$, $(34)\times(59)+$ $(58)$ | $\sigma_{e,n}$, $\sqrt{(60)^2+(61)^2-(60)\times(61)}$ |
| Values from elastic-stress calculation, table I (a). | 16 | 1301.0×10⁻⁶ | −3174.2 | −37,271 | −20,718 | −79,644 | 27,755 | 18,702 | −41,628 | 53,490 |
| | 17 | 1019.7 | −1538.9 | −28,938 | −25,257 | −106,070 | ------ | 16,412 | −67,593 | 77,120 |
| | 18 | 1407.2 | −1708.7 | −38,564 | −31,137 | −139,980 | ------ | 13,374 | −101,090 | 108,400 |
| | 19 | 1928.4 | −1909.8 | −49,881 | −39,567 | −181,250 | ------ | 9,592.7 | −141,720 | 146,750 |
| | $b$ | 2664.4 | −2053.3 | −58,543 | −50,232 | −213,380 | ------ | 4,595.2 | −174,750 | 177,090 |
| 1 | 17 | 999.55 | −1530.3 | −28,374 | −25,249 | −105,500 | 26,023 | 13,819 | −69,424 | 77,266 |
| | 18 | 744.61 | −1444.6 | −20,602 | −30,848 | −121,480 | ------ | 10,885 | −85,019 | 90,952 |
| | 19 | 691.00 | −1456.6 | −18,154 | −38,266 | −132,490 | ------ | 7,825.9 | −95,423 | 99,567 |
| | $b$ | 427.90 | −1376.0 | −9,780.7 | −46,810 | −124,080 | ------ | 4,595.8 | −87,861 | 90,247 |
| 2 | 17 | 999.45 | −1530.3 | −28,371 | −25,249 | −105,500 | 25,967 | 13,720 | −69,516 | 77,295 |
| | 18 | 729.23 | −1438.5 | −20,185 | −30,842 | −121,070 | ------ | 10,785 | −84,702 | 90,577 |
| | 19 | 691.60 | −1456.8 | −18,169 | −33,248 | −132,120 | ------ | 7,727.0 | −95,147 | 99,237 |
| | $b$ | 96.000 | −1275.5 | −2,544.3 | −46,680 | −116,540 | ------ | 4,595.5 | −80,413 | 82,807 |
| 3 | 17 | 999.45 | −1530.3 | −28,371 | −25,249 | −105,500 | 25,998 | 13,782 | −69,459 | 77,277 |
| | 18 | 744.46 | −1444.5 | −20,598 | −30,848 | −121,480 | ------ | 10,845 | −85,054 | 90,963 |
| | 19 | 682.10 | −1453.3 | −17,925 | −38,263 | −132,260 | ------ | 7,784.7 | −95,228 | 99,349 |
| | $b$ | 296.00 | −1336.1 | −6,904.0 | −46,761 | −121,010 | ------ | 4,595.4 | −84,826 | 87,215 |
| 4 | 17 | 999.45 | −1530.3 | −28,371 | −25,249 | −105,500 | 26,008 | 13,797 | −69,445 | 77,273 |
| | 18 | 734.30 | −1440.5 | −20,322 | −30,844 | −121,200 | ------ | 10,865 | −84,760 | 90,682 |
| | 19 | 691.90 | −1456.9 | −18,177 | −38,254 | −132,250 | ------ | 7,811.4 | −95,204 | 99,340 |
| | $b$ | 397.50 | −1366.8 | −9,117.9 | −46,781 | −123,220 | ------ | 4,595.2 | −87,022 | 89,408 |
| 5 | 17 | 999.45 | −1530.3 | −28,371 | −25,249 | −105,500 | [a] 26,006 | 13,794 | −69,448 | 77,274 |
| | 18 | 734.33 | −1440.5 | −20,323 | −30,844 | −121,200 | ------ | 10,862 | −84,763 | 90,683 |
| | 19 | 691.90 | −1456.9 | −18,177 | −38,254 | −132,250 | ------ | 7,907.8 | −95,207 | 99,341 |
| | $b$ | 382.00 | −1362.1 | −8,779.9 | −46,777 | −122,880 | ------ | 4,595.3 | −86,685 | 89,072 |

[a] This value of $\sigma_{t,a}$ is also substituted for the original value of $\sigma_{t,a}$ used in table I(a) to compute plastic stress for stations $a$ to 16.

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TABLE II.—ABRIDGED VALUES FROM CALCULATION OF STRESS DISTRIBUTION FOR EXAMPLE II

| $n$ | 1 | 2 | 5 | 6 | 7 | 25 | 25a | 25b | 38 | 39 | 54 | 60 | 61 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $r_n$ | $h_n$ | $E_n$ | $\alpha_n$ | $\Delta T_n$ | $H'_n$ | $[P'_n]+[Q'_n]$ | $H'_n-[P'_n]-[Q'_n]$ | $\sigma_{r,n}$ | $\sigma_{t,n}$ | $H'_n-[P'_n]-[Q'_n]-P'_n$ | $\sigma_{r,n}$ | $\sigma_{t,n}$ |
| a | 0.5000 | 4.3750 | 29.700×10⁶ | 8.5060×10⁻⁶ | 130 | ---------- | ---------- | ---------- | 36,093 | 36,093 | ---------- | 35,685 | 35,685 |
| 2 | .6250 | 4.3750 | 29.700 | 8.5060 | 130 | 0 ×10⁻⁶ | 0 ×10⁻⁶ | 0 ×10⁻⁶ | 36,037 | 36,069 | 0 ×10⁻⁶ | 35,629 | 35,661 |
| 3 | .7500 | 4.3750 | 29.700 | 8.5060 | 130 | 0 | 0 | 0 | 35,973 | 36,035 | 0 | 35,565 | 35,627 |
| 4 | 1.0000 | 4.3750 | 29.700 | 8.5060 | 130 | 0 | 0 | 0 | 35,814 | 35,944 | 0 | 35,406 | 35,536 |
| 5 | 1.2500 | 4.3750 | 29.700 | 8.5060 | 130 | 0 | 0 | 0 | 35,613 | 35,821 | 0 | 35,205 | 35,413 |
| 6 | 1.5000 | 4.3750 | 29.700 | 8.5060 | 130 | 0 | 0 | 0 | 35,370 | 35,675 | 0 | 34,962 | 35,267 |
| 7 | 2.0000 | 4.3750 | 29.700 | 8.5060 | 130 | 0 | 0 | 0 | 34,746 | 35,295 | 0 | 34,338 | 34,887 |
| 8 | 3.0000 | 3.8400 | 29.700 | 8.5060 | 130 | 0 | 0 | 0 | 37,240 | 36,215 | 0 | 36,781 | 35,783 |
| 9 | 4.0000 | 3.2750 | 29.700 | 8.5070 | 131 | 8.6400 | 0 | 8.6400 | 39,977 | 37,557 | 8.6400 | 39,461 | 37,056 |
| 10 | 5.0000 | 2.6800 | 29.700 | 8.5180 | 138 | 61.060 | 0 | 61.060 | 43,774 | 38,228 | 61.060 | 43,149 | 37,700 |
| 11 | 5.5000 | 2.8720 | 29.600 | 8.5330 | 147 | 78.870 | 0 | 78.870 | 46,415 | 37,649 | 78.870 | 45,720 | 37,083 |
| 12 | 6.0000 | 2.2100 | 29.600 | 8.5620 | 165 | 158.38 | 0 | 158.38 | 46,390 | 34,176 | 158.38 | 45,657 | 33,581 |
| 13 | 6.5000 | 2.1600 | 20.400 | 8.6130 | 197 | 284.03 | 0 | 284.03 | 43,627 | 20,320 | 284.03 | 42,887 | 25,710 |
| 14 | 7.0000 | 2.1550 | 29.200 | 8.7000 | 251 | 486.94 | 0 | 486.94 | 39,275 | 12,677 | 486.94 | 38,543 | 12,060 |
| 15 | 7.5000 | 2.7000 | 28.700 | 8.8410 | 339 | 813.40 | 0 | 813.40 | 26,535 | −12,109 | 813.40 | 25,954 | −12,673 |
| 16 | 8.0000 | 2.7000 | 27.000 | 9.0680 | 481 | 1364.6 | 0 | 1364.6 | 19,876 | −47,387 | 1364.6 | 19,290 | −47,041 |
| 17 | 8.2500 | 2.5500 | 27.300 | 9.2260 | 579 | 980.14 | 20.250 | 959.89 | 16,863 | −70,310 | 838.94 | 16,301 | −67,610 |
| 18 | 8.5000 | 2.3800 | 26.400 | 9.4200 | 700 | 1252.2 | 672.87 | 579.33 | 13,459 | −80,687 | 390.03 | 13,031 | −78,003 |
| 19 | 8.7500 | 2.1450 | 24.700 | 9.6570 | 848 | 1595.1 | 1236.5 | 358.60 | 10,192 | −81,665 | 251.64 | 9,988 | −72,108 |
| b | 9.0000 | 1.9100 | 20.700 | 9.9490 | 1030 | 2058.8 | 2282.4 | −224.10 | 7,062 | −61,549 | −105.13 | 7,062 | −60,196 |

TABLE III.—ABRIDGED VALUES FROM CALCULATION OF STRESS DISTRIBUTION FOR EXAMPLE III

| $n$ | 1 | 2 | 5 | 6 | 7 | 25 | 25a | 25b | 38 | 39 | 54 | 60 | 61 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $r_n$ | $h_n$ | $E_n$ | $\alpha_n$ | $\Delta T_n$ | $H'_n$ | $[P'_n]+[Q'_n]$ | $H'_n-[P'_n]-[Q'_n]$ | $\sigma_{r,n}$ | $\sigma_{t,n}$ | $H'_n-[P'_n]-[Q'_n]-P'_n$ | $\sigma_{r,n}$ | $\sigma_{t,n}$ |
| a | 0.5000 | 4.3750 | 28.800×10⁶ | 8.8260×10⁻⁶ | 330 | ---------- | ---------- | ---------- | 44,435 | 44,435 | ---------- | 44,418 | 44,418 |
| 2 | .6250 | 4.3750 | 28.800 | 8.8260 | 330 | 0 ×10⁻⁶ | 0 ×10⁻⁶ | 0 ×10⁻⁶ | 44,361 | 44,403 | 0 ×10⁻⁶ | 44,344 | 44,386 |
| 3 | .7500 | 4,3750 | 28.800 | 8.8260 | 330 | 0 | 0 | 0 | 44,277 | 44,359 | 0 | 44,260 | 44,342 |
| 4 | 1.0000 | 4.3750 | 28.800 | 8.8260 | 330 | 0 | 0 | 0 | 44,066 | 44,238 | 0 | 44,049 | 44,221 |
| 5 | 1.2500 | 4.3750 | 28.800 | 8.8260 | 330 | 0 | 0 | 0 | 43,802 | 44,079 | 0 | 43,785 | 44,062 |
| 6 | 1.5000 | 4.3750 | 28.800 | 8.8260 | 330 | 0 | 0 | 0 | 43,480 | 43,884 | 0 | 43,463 | 43,867 |
| 7 | 2.0000 | 4.3750 | 28.800 | 8.8260 | 330 | 0 | 0 | 0 | 42,655 | 43,381 | 0 | 42,638 | 43,364 |
| 8 | 3.0000 | 3.8400 | 28.800 | 8.8280 | 331 | 9.5000 | 0 | 9.5000 | 45,497 | 44,159 | 9.5000 | 45,478 | 44,141 |
| 9 | 4.0000 | 3.2750 | 28.700 | 8.8360 | 336 | 46.800 | 0 | 46.800 | 48,399 | 44,631 | 46.800 | 48,377 | 44,642 |
| 10 | 5.0000 | 2.6800 | 28.700 | 8.8620 | 352 | 150.50 | 0 | 150.50 | 52,198 | 43,481 | 150.50 | 52,172 | 43,459 |
| 11 | 5.5000 | 2.3720 | 28.600 | 8.8890 | 369 | 160.60 | 0 | 160.60 | 54,778 | 41,126 | 160.60 | 54,749 | 41,102 |
| 12 | 6.0000 | 2.2100 | 28.400 | 8.9310 | 395 | 247.70 | 0 | 247.70 | 54,062 | 35,558 | 247.70 | 54,032 | 35,533 |
| 13 | 6.5000 | 2.1600 | 28.200 | 8.9960 | 436 | 394.60 | 0 | 394.60 | 50,068 | 25,238 | 394.60 | 50,038 | 25,212 |
| 14 | 7.0000 | 2.1550 | 27.900 | 9.0980 | 466 | 587.80 | 0 | 587.80 | 44,243 | 9,660 | 587.80 | 44,212 | 9,634 |
| 15 | 7.5000 | 2.7000 | 27.300 | 9.2290 | 581 | 851.90 | 0 | 851.90 | 29,200 | −15,112 | 851.90 | 29,176 | −15,136 |
| 16 | 8.0000 | 2.7000 | 26.400 | 9.4200 | 700 | 1232.0 | 0 | 1232.0 | 21,352 | −44,828 | 1232.0 | 21,328 | −44,851 |
| 17 | 8.2500 | 2.5500 | 25.700 | 9.5400 | 775 | 799.50 | 141.19 | 658.31 | 18,041 | −58,588 | 633.84 | 18,026 | −57,981 |
| 18 | 8.5000 | 2.3800 | 24.500 | 9.6790 | 862 | 949.80 | 872.12 | 77.68 | 14,659 | −56,752 | 91.355 | 14,050 | −55,525 |
| 19 | 8.7500 | 2.1450 | 22.400 | 9.8410 | 963 | 1133.6 | 1343.5 | −209.90 | 11,807 | −44,465 | 200.34 | 11,807 | −44,478 |
| b | 9.0000 | 1.9100 | 19.300 | 10.029 | 1080 | 1354.1 | 2163.4 | −809.30 | 9,339 | −21,474 | −809.30 | 9,339 | −21,485 |

TABLE IV.—CALCULATION OF STRESS DISTRIBUTION FOR EXAMPLE IV

(a) Abridged values

| $n$ | 1 | 2 | 5 | 6 | 7 | 25 | 25a | 25b | 38 | 39 | 72 | 78 | 79 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $r_n$ | $h_n$ | $E_n$ | $\alpha_n$ | $\Delta T_n$ | $H'_n$ | $[P'_n]+[Q'_n]$ | $H'_n-[P'_n]-[Q'_n]$ | $\sigma_{r,n}$ | $\sigma_{t,n}$ | $H'_n-[P'_n]-[Q'_n]-Q'_n$ | $\sigma_{r,n}$ | $\sigma_{t,n}$ |
| a | 0.5000 | 4.3750 | 27.600×10⁶ | 9.1470×10⁻⁶ | 530 | 0 ×10⁻⁶ | 0 ×10⁻⁶ | ---------- | 46,006 | 46,006 | ---------- | 46,005 | 46,005 |
| 2 | .6250 | 4.3750 | 27.600 | 9.1470 | 530 | 0 | 0 | 0 ×10⁻⁶ | 45,932 | 45,974 | 0 ×10⁻⁶ | 45,931 | 45,973 |
| 3 | .7500 | 4.3750 | 27.600 | 9.1470 | 530 | 0 | 0 | 0 | 45,848 | 45,930 | 0 | 45,847 | 45,929 |
| 4 | 1.0000 | 4.3750 | 27.600 | 9.1470 | 530 | 0 | 0 | 0 | 45,637 | 45,809 | 0 | 45,636 | 45,808 |
| 5 | 1.2500 | 4.3750 | 27.600 | 9.1470 | 530 | 0 | 0 | 0 | 45,373 | 45,650 | 0 | 45,372 | 45,649 |
| 6 | 1.5000 | 4.3750 | 27.600 | 9.1470 | 530 | 0 | 0 | 0 | 45,051 | 45,455 | 0 | 45,050 | 45,454 |
| 7 | 2.0000 | 4.3750 | 27.600 | 9.1490 | 531 | 10.200 | 0 | 10.200 | 44,194 | 44,698 | 10.200 | 44,193 | 44,697 |
| 8 | 3.0000 | 3.8400 | 27.600 | 9.1580 | 537 | 59.700 | 0 | 59.700 | 46,959 | 44,419 | 59.700 | 46,958 | 44,418 |
| 9 | 4.0000 | 3.2750 | 27.500 | 9.1840 | 553 | 161.00 | 0 | 161.00 | 49,449 | 42,507 | 161.00 | 49,448 | 42,506 |
| 10 | 5.0000 | 2.6800 | 27.300 | 9.2380 | 587 | 343.90 | 0 | 343.90 | 52,329 | 37,155 | 343.90 | 52,328 | 37,154 |
| 11 | 5.5000 | 2.3720 | 27.100 | 9.2820 | 614 | 276.40 | 0 | 276.40 | 54,197 | 32,627 | 276.40 | 54,195 | 32,626 |
| 12 | 6.0000 | 2.2100 | 26.800 | 9.3380 | 649 | 361.30 | 0 | 361.30 | 52,668 | 25,030 | 361.30 | 52,666 | 25,028 |
| 13 | 6.5000 | 2.1600 | 26.500 | 9.4080 | 693 | 459.30 | 0 | 459.30 | 47,920 | 14,400 | 459.30 | 47,918 | 14,399 |
| 14 | 7.0000 | 2.1550 | 25.900 | 9.5000 | 750 | 605.30 | 0 | 605.30 | 41,517 | 227 | 605.30 | 41,515 | 227 |
| 15 | 7.5000 | 2.7000 | 25.100 | 9.6100 | 819 | 745.60 | 0 | 745.60 | 26,780 | −19,203 | 745.60 | 26,778 | −19,204 |
| 16 | 8.0000 | 2.7000 | 23.800 | 9.7480 | 905 | 951.30 | 0 | 951.30 | 19,146 | −36,527 | 951.30 | 19,145 | −36,528 |
| 17 | 8.2500 | 2.5500 | 22.700 | 9.8870 | 954 | 705.70 | 166.17 | 539.53 | 16,107 | −45,462 | 538.54 | 16,105 | −45,443 |
| 18 | 8.5000 | 2.3800 | 21.200 | 9.9120 | 1007 | 453.80 | 858.44 | −404.64 | 13,200 | −32,446 | −405.17 | 13,199 | −32,418 |
| 19 | 8.7500 | 2.1450 | 19.800 | 10.007 | 1066 | 685.60 | 1334.0 | −648.40 | 11,002 | −16,554 | −647.94 | 11,003 | −16,530 |
| b | 9.0000 | 1.9100 | 17.900 | 10.109 | 1130 | 756.00 | 2163.4 | −1407.4 | 9,340 | 10,470 | −1406.5 | 9,334 | 10,470 |

TABLE IV.—CALCULATION OF STRESS DISTRIBUTION FOR EXAMPLE IV—Concluded

(b) Calculation of effect of creep on final stresses

| $n$ | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 |
|---|---|---|---|---|---|---|---|---|---|
| | $c_n$ | $\Gamma_n$, (63)×5 | $\delta_{r,n}$, $\frac{(64)}{(41)\times 2}\times$ $[2\times(38)-(39)]$ | $\delta_{t,n}$, $\frac{(64)}{(41)\times 2}\times$ $[2\times(39)-(38)]$ | (65)×(42) | $(65)_{n-1}$×(43) | (66)×(44) | $(66)_{n-1}$×(45) | $Q'_n$, (67)+(68)−(69)+(70) |
| 17 | 0.2×10⁻⁶ | 1.0×10⁻⁶ | 0.70226×10⁻⁶ | −0.96766×10⁻⁶ | 0.010641×10⁻⁶ | 0.00000×10⁻⁶ | −0.98237×10⁻⁶ | 0.00000×10⁻⁶ | 0.99301×10⁻⁶ |
| 18 | .3 | 1.5 | 1.0848 | −1.4396 | .015953 | .010641 | −1.4608 | −.95300 | .53439 |
| 19 | .2 | 1.0 | .80246 | −.91800 | .011464 | .015953 | −.93113 | −1.4184 | −.45985 |
| b | 0 | 0 | 0 | 0 | 0 | .011464 | 0 | −.90488 | −.89342 |

| $n$ | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 |
|---|---|---|---|---|---|---|---|---|
| | $H'_n-[P'_n]-[Q'_n]-Q'_n$, (25b)−(71) | $M_n$, [(72)×(10)+(14)×(21)]+(26) | $M'_n$, [(72)×(8)+(14)×(20)]+(26) | $B_{r,n}$, $(27)\times(75)_{n-1}+(28)\times(76)_{n-1}+(73)$ | $B_{t,n}$, $(29)\times(75)_{n-1}+(30)\times(76)_{n-1}+(74)$ | $\sigma_{t,a}$, $[\sigma_{r,b}-(75)_b]\div(33)_b$ | $\sigma_{r,n}$, (33)×(77)+(75) | $\sigma_{t,n}$, (34)×(77)+(76) |
| 17 | 538.54×10⁻⁶ | −2469.1 | −12,876 | −52,373 | −105,190 | 46,005 | 16,105 | −45,443 |
| 18 | −405.17 | −2247.2 | 7,607.5 | −59,834 | −91,828 | ------ | 13,199 | −32,418 |
| 19 | −647.94 | −2306.2 | 11,751 | −69,520 | −76,973 | ------ | 11,003 | −16,536 |
| b | −1400.5 | −2230.7 | 23,905 | −80,379 | −50,965 | ------ | 9,334 | 10,470 |

TABLE V.—ABRIDGED VALUES FROM CALCULATION OF STRESS DISTRIBUTION FOR EXAMPLE V

| $n$ | 1 | 2 | 5 | 6 | 7 | 25 | 25a | 25b | 38 | 39 | 54 | 60 | 61 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $r_n$ | $h_n$ | $E_n$ | $\alpha_n$ | $\Delta T_n$ | $H'_n$ | $[P'_n]+[Q'_n]$ | $H'_n-[P'_n]-[Q'_n]$ | $\sigma_{r,n}$ | $\sigma_{t,n}$ | $H'_n-[P'_n]-[Q'_n]-P'_n$ | $\sigma_{r,n}$ | $\sigma_{t,n}$ |
| a | 0.5000 | 4.3750 | 28.800×10⁶ | 8.8260×10⁻⁶ | 331 | ------ | ------ | ------ | 22,285 | 22,285 | ------ | 22,285 | 22,285 |
| 2 | .6250 | 4.3750 | 28.800 | 8.8300 | 332 | 9.49×10⁻⁶ | 0 ×10⁻⁶ | 9.49×10⁻⁶ | 22,253 | 22,023 | 9.49×10⁻⁶ | 22,253 | 22,023 |
| 3 | .7500 | 4.3750 | 28.800 | 8.8310 | 333 | 9.16 | 0 | 9.16 | 22,181 | 21,811 | 9.16 | 22,181 | 21,811 |
| 4 | 1.0000 | 4.3750 | 28.800 | 8.8330 | 334 | 9.50 | 0 | 9.50 | 22,027 | 21,645 | 9.50 | 22,027 | 21,645 |
| 5 | 1.2500 | 4.3750 | 28.700 | 8.8390 | 338 | 37.36 | 0 | 37.36 | 21,801 | 20,687 | 37.36 | 21,801 | 20,687 |
| 6 | 1.5000 | 4.3750 | 28.700 | 8.8440 | 341 | 28.22 | 0 | 28.22 | 21,501 | 20,085 | 28.22 | 21,501 | 20,085 |
| 7 | 2.0000 | 4.3750 | 28.700 | 8.8580 | 350 | 84.50 | 0 | 84.50 | 20,744 | 18,160 | 84.50 | 20,744 | 18,160 |
| 8 | 3.0000 | 3.8400 | 28.500 | 8.8970 | 374 | 226.70 | 0 | 226.70 | 21,090 | 14,149 | 226.70 | 21,090 | 14,149 |
| 9 | 4.0000 | 3.2750 | 28.300 | 8.9530 | 409 | 334.78 | 0 | 334.78 | 20,964 | 8,300 | 334.78 | 20,964 | 8,300 |
| 10 | 5.0000 | 2.6800 | 28.100 | 9.0240 | 453 | 426.09 | 0 | 426.09 | 20,680 | 1,010 | 426.09 | 20,680 | 1,010 |
| 11 | 5.5000 | 2.3720 | 28.000 | 9.0650 | 479 | 254.27 | 0 | 254.27 | 20,585 | −3,342 | 254.27 | 20,585 | −3,342 |
| 12 | 6.0000 | 2.2100 | 27.800 | 9.1120 | 508 | 287.76 | 0 | 287.76 | 19,144 | −8,724 | 287.76 | 19,144 | −8,724 |
| 13 | 6.5000 | 2.1600 | 27.600 | 9.1620 | 539 | 309.42 | 0 | 309.42 | 16,529 | −14,887 | 309.42 | 16,529 | −14,887 |
| 14 | 7.0000 | 2.1550 | 27.400 | 9.2140 | 572 | 332.09 | 0 | 332.09 | 13,400 | −21,552 | 332.09 | 13,400 | −21,552 |
| 15 | 7.5000 | 2.7000 | 27.100 | 9.2720 | 608 | 366.97 | 0 | 366.97 | 7,748 | −29,867 | 366.97 | 7,748 | −29,867 |
| 16 | 8.0000 | 2.7000 | 26.800 | 9.3330 | 646 | 391.74 | 0 | 391.74 | 4,365 | −37,781 | 391.74 | 4,365 | −37,781 |
| 17 | 8.2500 | 2.5500 | 26.700 | 9.3650 | 666 | 207.97 | 167.16 | 40.840 | 2,879 | −37,429 | 40.840 | 2,879 | −37,429 |
| 18 | 8.5000 | 2.3800 | 26.500 | 9.3990 | 687 | 220.02 | 858.97 | −638.97 | 1,675 | −19,393 | −638.97 | 1,675 | −19,393 |
| 19 | 8.7500 | 2.1450 | 26.300 | 9.4490 | 708 | 232.78 | 1333.5 | −1100.5 | 1,171 | 9,833 | −1100.5 | 1,171 | 9,833 |
| b | 9.0000 | 1.9100 | 26.100 | 9.4990 | 730 | 244.38 | 2162.5 | −1918.5 | 1,764 | 58,858 | −1918.5 | 1,764 | 58,858 |

TABLE VI.—ABRIDGED VALUES FROM CALCULATION OF STRESS DISTRIBUTION FOR EXAMPLE VI

| $n$ | 1 | 2 | 5 | 6 | 7 | 25 | 25a | 25b | 38 | 39 | 54 | 60 | 61 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $r_n$ | $h_n$ | $E_n$ | $\alpha_n$ | $\Delta T_n$ | $H'_n$ | $[P'_n]+[Q'_n]$ | $H'_n-[P'_n]-[Q'_n]$ | $\sigma_{r,n}$ | $\sigma_{t,n}$ | $H'_n-[P'_n]-[Q'_n]-P'_n$ | $\sigma_{r,n}$ | $\sigma_{t,n}$ |
| a | 0.5000 | 4.3750 | 29.300×10⁶ | 8.6760×10⁻⁶ | 236 | ------ | ------ | ------ | 3616 | 3,616 | ------ | 3750 | 3,750 |
| 2 | .6250 | 4.3750 | 29.300 | 8.6770 | 237 | 8.91×10⁻⁶ | 0 ×10⁻⁶ | 8.91×10⁻⁶ | 3592 | 3,375 | 8.91×10⁻⁶ | 3726 | 3,509 |
| 3 | .7500 | 4.3750 | 29.300 | 8.6790 | 238 | 9.15 | 0 | 9.15 | 3537 | 3,160 | 9.15 | 3672 | 3,294 |
| 4 | 1.0000 | 4.3750 | 29.300 | 8.6840 | 241 | 27.20 | 0 | 27.20 | 3364 | 2,529 | 27.20 | 3498 | 2,662 |
| 5 | 1.2500 | 4.3750 | 29.200 | 8.6880 | 244 | 27.03 | 0 | 27.03 | 3140 | 1,956 | 27.03 | 3274 | 2,090 |
| 6 | 1.5000 | 4.3750 | 29.200 | 8.6930 | 247 | 27.30 | 0 | 27.30 | 2897 | 1,402 | 27.30 | 3030 | 1,536 |
| 7 | 2.0000 | 4.3750 | 29.200 | 8.7010 | 252 | 45.48 | 0 | 45.48 | 2417 | 557 | 45.48 | 2551 | 690 |
| 8 | 3.0000 | 3.8400 | 29.100 | 8.7190 | 263 | 100.44 | 0 | 100.44 | 1725 | −1,301 | 100.44 | 1876 | −1,160 |
| 9 | 4.0000 | 3.2750 | 29.100 | 8.7370 | 274 | 100.84 | 0 | 100.84 | 933 | −3,144 | 100.84 | 1106 | −2,990 |
| 10 | 5.0000 | 2.6800 | 29.000 | 8.7560 | 286 | 110.28 | 0 | 110.28 | 2 | −5,260 | 110.28 | 207 | −5,087 |
| 11 | 5.5000 | 2.3720 | 29.000 | 8.7640 | 291 | 46.11 | 0 | 46.11 | −545 | −6,093 | 46.11 | −317 | −5,907 |
| 12 | 6.0000 | 2.2100 | 28.900 | 8.7730 | 297 | 55.26 | 0 | 55.26 | −1108 | −7,185 | 55.26 | −868 | −6,991 |
| 13 | 6.5000 | 2.1600 | 28.900 | 8.7820 | 302 | 46.58 | 0 | 46.58 | −1639 | −8,043 | 46.58 | −1396 | −7,843 |
| 14 | 7.0000 | 2.1550 | 28.900 | 8.7910 | 308 | 55.46 | 0 | 55.46 | −2140 | −9,150 | 55.46 | −1900 | −8,947 |
| 15 | 7.5000 | 2.7000 | 28.900 | 8.7990 | 313 | 46.46 | 0 | 46.46 | −2165 | −9,819 | 46.46 | −1974 | −9,631 |
| 16 | 8.0000 | 2.7000 | 28.800 | 8.8090 | 319 | 55.98 | 0 | 55.98 | −2676 | −10,889 | 55.98 | −2486 | −10,702 |
| 17 | 8.2500 | 2.5500 | 28.800 | 8.8140 | 322 | 28.04 | 167.16 | −139.12 | −3025 | −6,756 | −139.12 | −2824 | −6,564 |
| 18 | 8.5000 | 2.3800 | 28.800 | 8.8170 | 324 | 18.60 | 858.97 | −840.37 | −3000 | 17,132 | −840.37 | −2785 | 17,329 |
| 19 | 8.7500 | 2.1450 | 28.800 | 8.8220 | 327 | 28.09 | 1333.5 | −1305.4 | −2197 | 53,534 | −1305.4 | −1959 | 53,740 |
| b | 9.0000 | 1.9100 | 28.800 | 8.8260 | 330 | 27.79 | 2162.5 | −2134.7 | 0 | 112,600 | −1461.0 | 0 | 93,672 |

TABLE VII.—ABRIDGED VALUES FROM CALCULATION OF STRESS DISTRIBUTION FOR EXAMPLE VII

| $n$ | 1 | 2 | 5 | 6 | 7 | 25 | 25a | 25b | 38 | 39 | 54 | 60 | 61 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $r_n$ | $h_n$ | $E_n$ | $\alpha_n$ | $\Delta T_n$ | $H'_n$ | $[P'_n]+[Q'_n]$ | $H'_n-[P'_n]-[Q'_n]$ | $\sigma_{r,n}$ | $\sigma_{t,n}$ | $H'_n-[P'_n]-[Q'_n]-P'_n$ | $\sigma_{r,n}$ | $\sigma_{t,n}$ |
| a | 0.5000 | 4.3750 | Constant at 30.000×10⁶ | Constant at 8.2970×10⁻⁶ | Constant at 0 | Constant at 0 | ------ | ------ | −2468 | −2,468 | ------ | −2356 | −2,356 |
| 2 | .6250 | 4.3750 | | | | | 0 ×10⁻⁶ | 0 ×10⁻⁶ | −2468 | −2,408 | 0 ×10⁻⁶ | −2356 | −2,356 |
| 3 | .7500 | 4.3750 | | | | | 0 | 0 | −2468 | −2,468 | 0 | −2356 | −2,356 |
| 4 | 1.0000 | 4.3750 | | | | | 0 | 0 | −2468 | −2,468 | 0 | −2356 | −2,356 |
| 5 | 1.2500 | 4.3750 | | | | | 0 | 0 | −2468 | −2,468 | 0 | −2356 | −2,356 |
| 6 | 1.5000 | 4.3750 | | | | | 0 | 0 | −2468 | −2,468 | 0 | −2356 | −2,356 |
| 7 | 2.0000 | 4.3750 | | | | | 0 | 0 | −2468 | −2,468 | 0 | −2356 | −2,356 |
| 8 | 3.0000 | 3.8400 | | | | | 0 | 0 | −2779 | −2,614 | 0 | −2653 | −2,490 |
| 9 | 4.0000 | 3.2750 | | | | | 0 | 0 | −3183 | −2,849 | 0 | −3039 | −2,720 |
| 10 | 5.0000 | 2.6800 | | | | | 0 | 0 | −3779 | −3,193 | 0 | −3608 | −3,049 |
| 11 | 5.5000 | 2.3720 | | | | | 0 | 0 | −4202 | −3,428 | 0 | −4012 | −3,273 |
| 12 | 6.0000 | 2.2100 | | | | | 0 | 0 | −4437 | −3,604 | 0 | −4237 | −3,442 |
| 13 | 6.5000 | 2.1600 | | | | | 0 | 0 | −4475 | −3,705 | 0 | −4273 | −3,537 |
| 14 | 7.0000 | 2.1550 | | | | | 0 | 0 | −4432 | −3,762 | 0 | −4232 | −3,592 |
| 15 | 7.5000 | 2.7000 | | | | | 0 | 0 | −3518 | −3,476 | 0 | −3359 | −3,319 |
| 16 | 8.0000 | 2.7000 | | | | | 0 | 0 | −3515 | −3,479 | 0 | −3356 | −3,321 |
| 17 | 8.2500 | 2.5500 | | | | | 167.16 | −167.16 | −3644 | 1,358 | −167.16 | −3476 | 1,519 |
| 18 | 8.5000 | 2.3800 | | | | | 858.97 | −858.97 | −3378 | 26,512 | −858.97 | −3199 | 26,077 |
| 19 | 8.7500 | 2.1450 | | | | | 1333.5 | −1333.5 | −2292 | 65,023 | −1333.5 | −2093 | 65,195 |
| b | 9.0000 | 1.9100 | | | | | 1488.8 | −1488.8 | 0 | 107,180 | −947.81 | 0 | 91,109 |

TABLE VIII.—CALCULATION OF STRESS DISTRIBUTION FOR PARALLEL-SIDED DISK

(a) Abridged values

| $n$ | 1 | 2 | 5 | 6 | 7 | 25 | 25a | 25b | 38 | 39 | 40 | 41 | 60 | 61 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $r_n$ | $h_n$ | $E_n$ | $\alpha_n$ | $\Delta T_n$ | $H'_n$ | $[P'_n]+[Q'_n]$ | $H'_n-[P'_n]-[Q'_n]$ | $\sigma_{r,n}$ | $\sigma_{t,n}$ | $\sigma_{y,n}$ | $\sigma_{e,n}$ | $\sigma_{r,n}$ | $\sigma_{t,n}$ |
| a | 0.5000 | Constant at 1.0000 | Constant at 30.000×10⁶ | Constant at 8.2970×10⁻⁶ | Constant at 0 | Constant at 0 | Constant at 0 | Constant at 0 | 0 | 176,040 | Constant at 100,000 | 176,040 | 0 | 124,520 |
| 2 | .6250 | | | | | | | | 31,650 | 142,520 | | 129,620 | 24,062 | 127,150 |
| 3 | .7500 | | | | | | | | 48,400 | 124,850 | | 109,030 | 41,519 | 124,510 |
| 4 | 1.0000 | | | | | | | | 64,573 | 100,350 | | 92,802 | 60,031 | 111,630 |
| 5 | 1.5000 | | | | | | | | 74,394 | 91,900 | | 84,518 | 72,549 | 93,977 |
| 6 | 2.0000 | | | | | | | | 75,607 | 86,678 | | 81,704 | 74,625 | 87,813 |
| 7 | 2.5000 | | | | | | | | 74,187 | 83,268 | | 79,119 | 73,589 | 84,005 |
| 8 | 3.0000 | | | | | | | | 71,369 | 80,278 | | 76,215 | 70,976 | 80,803 |
| 9 | 3.5000 | | | | | | | | 67,553 | 77,268 | | 72,898 | 67,282 | 77,669 |
| 10 | 4.0000 | | | | | | | | 62,892 | 74,060 | | 69,156 | 62,700 | 74,390 |
| 11 | 4.5000 | | | | | | | | 57,471 | 70,579 | | 65,024 | 57,332 | 70,845 |
| 12 | 5.0000 | | | | | | | | 51,322 | 66,782 | | 60,551 | 51,223 | 67,009 |
| 13 | 5.5000 | | | | | | | | 44,473 | 62,650 | | 55,827 | 44,401 | 62,847 |
| 14 | 6.0000 | | | | | | | | 36,934 | 58,163 | | 50,979 | 36,883 | 58,339 |
| 15 | 6.5000 | | | | | | | | 28,711 | 53,817 | | 46,219 | 28,678 | 53,476 |
| 16 | 7.0000 | | | | | | | | 19,813 | 48,103 | | 41,873 | 19,792 | 48,249 |
| 17 | 7.5000 | | | | | | | | 10,248 | 42,518 | | 38,433 | 10,232 | 42,654 |
| b | 8.0000 | | | | | | | | 0 | 36,562 | | 36,562 | 0 | 36,919 |

(b) Calculation for first approximation of plastic-stress distribution

| $n$ | 42 | 43 | 44 | 45 | 46 | 46a | 47 | 48 | 49 | 50 | 51 | 52 | 53 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (9)/(1) | $(9)/(1)_{n-1}$ | 1+(42) | 1−(43) | $\epsilon_{p,n}$ (graph reading, fig. 7) | $\epsilon_{p,n}$ (estimate) | $\Delta_{r,n}$, $\frac{(46a)}{(41)\times 2}\times[2\times(38)-(39)]$ | $\Delta_{t,n}$, $\frac{(46a)}{(41)\times 2}\times[2\times(39)-(38)]$ | (47)×(42) | $(47)_{n-1}\times(43)$ | (48)×(44) | $(48)_{n-1}\times(45)$ | $P'_n$, (49)+(50)−(51)+(52) |
| a | ------ | ------ | ------ | ------ | 1800×10⁻⁶ | 1800×10⁻⁶ | −900.00×10⁻⁶ | 1800.00×10⁻⁶ | ------ | ------ | ------ | ------ | ------ |
| 2 | 0.10000 | 0.12500 | 1.1000 | 0.87500 | 550 | 550 | −168.06 | 587.56 | −16.806×10⁻⁶ | −112.50×10⁻⁶ | 591.32×10⁻⁶ | 1575.0×10⁻⁶ | 854.37×10⁻⁶ |
| 3 | .08333 | .10000 | 1.0833 | .90000 | 300 | 300 | −38.59 | 276.95 | −3.2158 | −16.806 | 300.02 | 483.80 | 163.75 |
| 4 | .12500 | .16670 | 1.1250 | .83333 | 0 | 0 | 0 | 0 | 0 | −6.4330 | 0 | 230.78 | 224.34 |

| $n$ | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 |
|---|---|---|---|---|---|---|---|---|---|
| | $H'_n-[P'_n]-[Q'_n]-P'_n$, (25b)−(53) | $M_n$, [(54)×(10)+(14)×(21)]+(26) | $M'_n$, [(54)×(8)+(14)×(20)]+(26) | $B_{r,n}$, $(27)\times(57)_{n-1}+(28)\times(58)_{n-1}+(55)$ | $B_{t,n}$, $(29)\times(57)_{n-1}+(30)\times(58)_{n-1}+(56)$ | $\sigma_{t,a}$, $[\sigma_{r,b}-(57)_b]+(33)_b$ | $\sigma_{r,n}$, (33)×(59)+(57) | $\sigma_{t,n}$, (34)×(59)+(58) | $\sigma_{e,n}$, $\sqrt{(60)^2+(61)^2-(60)\times(61)}$ |
| a | ------ | ------ | ------ | ------ | ------ | Constant at 138,510 | ------ | ------ | ------ |
| 2 | −854.37×10⁻⁶ | 2,144.1 | 23,499 | 2,144.1 | 23,499 | | 0 | 138,510 | 138,510 |
| 3 | −163.75 | 117.81 | 4,475.8 | 5,589.1 | 24,590 | | 27,216 | 135,700 | 124,390 |
| 4 | −224.34 | 101.23 | 5,829.7 | 9,835.0 | 25,875 | | 43,981 | 122,980 | 107,030 |
| 5 | 0 | −1,872.7 | −795.28 | 12,435 | 19,838 | | 61,477 | 109,960 | 95,450 |
| 6 | 0 | −2,641.2 | −1,050.8 | 11,424 | 17,021 | | ------ | ------ | ------ |
| 7 | 0 | −3,423.3 | −1,302.6 | 9,013.8 | 14,655 | | ------ | ------ | ------ |
| 8 | 0 | −4,212.0 | −1,551.8 | 5,667.5 | 12,209 | | ------ | ------ | ------ |
| 9 | 0 | −5,005.0 | −1,799.2 | 1,533.5 | 9,519.1 | | ------ | ------ | ------ |
| 10 | 0 | −5,800.2 | −2,045.1 | −3,328.3 | 6,519.0 | | ------ | ------ | ------ |
| 11 | 0 | −6,597.3 | −2,290.8 | −8,889.8 | 3,178.3 | | ------ | ------ | ------ |
| 12 | 0 | −7,395.5 | −2,534.7 | −15,136 | −518.0 | | ------ | ------ | ------ |
| 13 | 0 | −8,194.6 | −2,778.5 | −22,060 | −4,578.8 | | ------ | ------ | ------ |
| 14 | 0 | −8,994.4 | −3,021.9 | −29,657 | −9,009.6 | | ------ | ------ | ------ |
| 15 | 0 | −9,794.6 | −3,265.0 | −37,923 | −13,814 | | ------ | ------ | ------ |
| 16 | 0 | −10,595 | −3,507.8 | −46,856 | −18,994 | | ------ | ------ | ------ |
| 17 | 0 | −11,396 | −3,750.1 | −56,454 | −24,550 | | ------ | ------ | ------ |
| b | 0 | −12,197 | −3,992.5 | −66,717 | −30,483 | | ------ | ------ | ------ |